Pressure and Manometers

1.1

What will be the (a) the gauge pressure and (b) the absolute pressure of water at depth 12m below the surface? ρ_{water} = 1000 kg/m^3, and p $_{atmosphere}$ = 101kN/m^2.
[117.72 kN/m^2, 218.72 kN/m^2]

a)

$$\begin{aligned} p_{gauge} &= \rho g h \\ &= 1000 \times 9.81 \times 12 \\ &= 117\,720 \ N/m^2, (Pa) \\ &= 117.7 \ kN/m^2, (kPa) \end{aligned}$$

b)

$$\begin{aligned} p_{absolute} &= p_{gauge} + p_{atmospheric} \\ &= (117\,720 + 101) \ N/m^2, (Pa) \\ &= 218.7 \ kN/m^2, (kPa) \end{aligned}$$

1.2
At what depth below the surface of oil, relative density 0.8, will produce a pressure of 120 kN/m^2? What depth of water is this equivalent to?
[15.3m, 12.2m]

a)

$$\begin{aligned} \rho &= \gamma \rho_{water} \\ &= 0.8 \times 1000 \ kg/m^3 \\ p &= \rho g h \\ h &= \frac{p}{\rho g} = \frac{120 \times 10^3}{800 \times 9.81} = 15.29 m \text{ of oil} \end{aligned}$$

b)

$$\begin{aligned} \rho &= 1000 \ kg/m^3 \\ h &= \frac{120 \times 10^3}{1000 \times 9.81} = 12.23 m \text{ of water} \end{aligned}$$

1.3
What would the pressure in kN/m^2 be if the equivalent head is measured as 400mm of (a) mercury γ=13.6 (b) water (c) oil specific weight 7.9 kN/m^3 (d) a liquid of density 520 kg/m^3?
[53.4 kN/m^2, 3.92 kN/m^2, 3.16 kN/m^2, 2.04 kN/m^2]

a)

$$\begin{aligned} \rho &= \gamma \rho_{water} \\ &= 13.6 \times 1000 \ kg/m^3 \\ p &= \rho g h \\ &= \left(13.6 \times 10^3\right) \times 9.81 \times 0.4 = 53366 \ N/m^2 \end{aligned}$$

b)

$$\begin{aligned} p &= \rho g h \\ &= \left(10^3\right)\times 9.81\times 0.4 = 3924\ N/m^2 \end{aligned}$$

c)

$$\begin{aligned} \omega &= \rho g \\ p &= \rho g h \\ &= \left(7.9\times 10^3\right)\times 0.4 = 3160\ N/m^2 \end{aligned}$$

d)

$$\begin{aligned} p &= \rho g h \\ &= 520\times 9.81\times 0.4 = 2040\,N/m^2 \end{aligned}$$

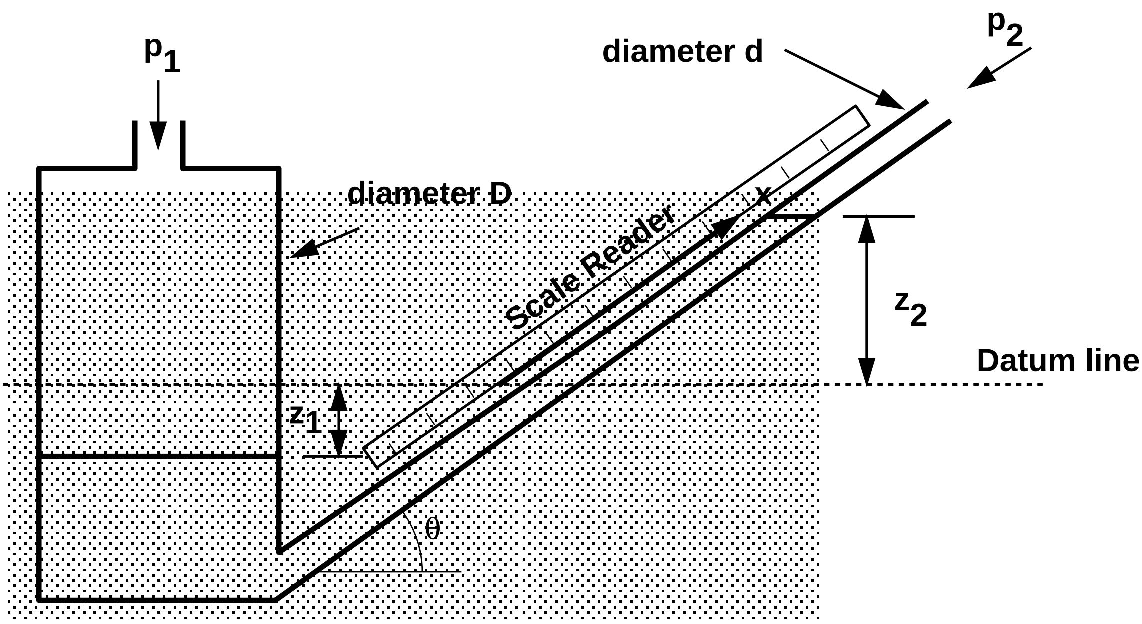
1.4
A manometer connected to a pipe indicates a negative gauge pressure of 50mm of mercury. What is the absolute pressure in the pipe in Newtons per square metre is the atmospheric pressure is 1 bar?
[93.3 kN/m^2]

$$\begin{aligned} p_{atmosphere} &= 1bar = 1\times 10^5\ N/m^2 \\ p_{absolute} &= p_{gauge} + p_{atmospheric} \\ &= \rho g h + p_{atmospheric} \\ &= -13.6\times 10^3\times 9.81\times 0.05 + 10^5\ \ N/m^2,\ (Pa) \\ &= 93.33\ kN/m^2,\ (kPa) \end{aligned}$$

1.5
What height would a water barometer need to be to measure atmospheric pressure?
[>10m]

$$\begin{aligned} p_{atmosphere} &\approx 1bar = 1\times 10^5\ N/m^2 \\ 10^5 &= \rho g h \\ h &= \frac{10^5}{1000\times 9.81} = 10.19\,m\ of\ \ water \\ h &= \frac{10^5}{(13.6\times 10^3)\times 9.81} = 0.75m\ of\ \ mercury \end{aligned}$$

1.6
An inclined manometer is required to measure an air pressure of 3mm of water to an accuracy of +/- 3%. The inclined arm is 8mm in diameter and the larger arm has a diameter of 24mm. The manometric fluid has density 740 kg/m^3 and the scale may be read to +/- 0.5mm.
What is the angle required to ensure the desired accuracy may be achieved?
[12° 39']



$$p_1 - p_2 = \rho_{man} gh = \rho_{man} g(z_1 + z_2)$$

Volume moved from left to right $= z_1 A_1 = \dfrac{z_2}{\sin\theta} A_2 = x A_2$

$$= z_1 \frac{\pi D^2}{4} = \frac{z_2}{\sin\theta} \frac{\pi d^2}{4} = x \frac{\pi d^2}{4}$$

$$z_1 = \frac{z_2}{\sin\theta} \frac{d^2}{D^2} = x \frac{d^2}{D^2}$$

$$p_1 - p_2 = \rho_{man} gx \left(\sin\theta + \frac{d^2}{D^2} \right)$$

$$\rho_{water} gh = \rho_{man} gx \left(\sin\theta + \frac{d^2}{D^2} \right)$$

$$\rho_{water} gh = (0.74 \times \rho_{water}) gx \left(\sin\theta + \frac{0.008^2}{0.024^2} \right)$$

$$h = 0.74\ x\ (\sin\theta + 0.1111)$$

The head being measured is 3% of 3mm = 0.003x0.03 = 0.00009m

This 3% represents the smallest measurement possible on the manometer, 0.5mm = 0.0005m, giving

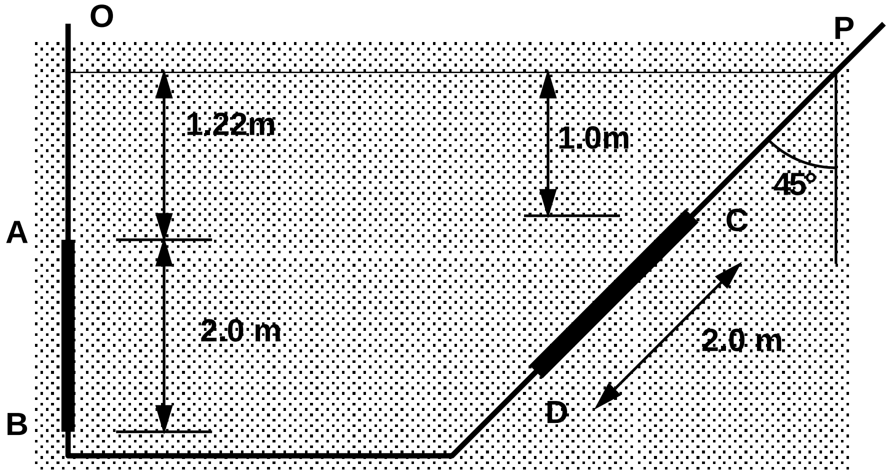
$$0.00009 = 0.74 \times 0.0005\ (\sin\theta + 0.1111)$$

$$\sin\theta = 0.132$$

$$\theta = 7.6^\circ$$

[This is <u>not</u> the same as the answer given on the question sheet]

1.7
Determine the resultant force due to the water acting on the 1m by 2m rectangular area AB shown in the diagram below.
[43 560 N, 2.37m from O]



The magnitude of the resultant force on a submerged plane is:

R = pressure at <u>centroid</u> $\times$ area of surface

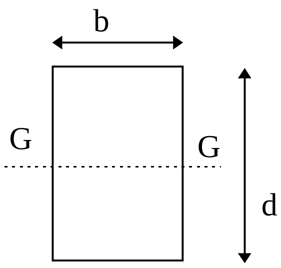
$$R = \rho g \bar{z} A$$
$$= 1000 \times 9.81 \times (1.22 + 1) \times (1 \times 2)$$
$$= 43\,556 \; N/m^2$$

This acts at right angle to the surface through the <u>centre of pressure</u>.

$$Sc = \frac{I_{OO}}{A\bar{x}} = \frac{\text{2nd moment of area about a line through O}}{\text{1st moment of area about a line through O}}$$

By the parallel axis theorem (which will be given in an exam), $I_{oo} = I_{GG} + A\bar{x}^2$, where I_{GG} is the 2nd moment of area about a line through the centroid and can be found in tables.

$$Sc = \frac{I_{GG}}{A\bar{x}} + \bar{x}$$

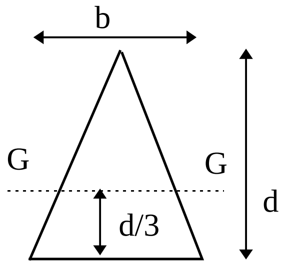


For a rectangle $I_{GG} = \dfrac{bd^3}{12}$

As the wall is vertical, $Sc = D$ and $\bar{x} = \bar{z}$,

$$Sc = \frac{1 \times 2^3}{12(1 \times 2)(1.22 + 1)} + (1.22 + 1)$$
$$= 2.37 \; m \text{ from O}$$

1.8
Determine the resultant force due to the water acting on the 1.25m by 2.0m triangular area CD shown in the figure above. The apex of the triangle is at C.
[43.5×10^3N, 2.821m from P]



For a triangle $I_{GG} = \dfrac{bd^3}{36}$

Depth to centre of gravity is $\bar{z} = 1.0 + 2\dfrac{2}{3}\cos 45 = 1.943m$.

$$
\begin{aligned}
R &= \rho g \bar{z} A \\
&= 1000 \times 9.81 \times 1.943 \times \left(\frac{2.0 \times 1.25}{2.0}\right) \\
&= 23826 \ N/m^2
\end{aligned}
$$

Distance from P is $\bar{x} = \bar{z} / \cos 45 = 2.748m$

Distance from P to centre of pressure is

$$
\begin{aligned}
Sc &= \frac{I_{oo}}{A\bar{x}} \\
I_{oo} &= I_{GG} + A\bar{x}^2 \\
Sc &= \frac{I_{GG}}{A\bar{x}} + \bar{x} = \frac{1.25 \times 2^3}{36(1.25)(2.748)} + (2.748) \\
&= 2.829m
\end{aligned}
$$

Forces on submerged surfaces

2.1

Obtain an expression for the depth of the centre of pressure of a plane surface wholly submerged in a fluid and inclined at an angle to the free surface of the liquid.
A horizontal circular pipe, 1.25m diameter, is closed by a butterfly disk which rotates about a horizontal axis through its centre. Determine the torque which would have to be applied to the disk spindle to keep the disk closed in a vertical position when there is a 3m head of fresh water above the axis.
[1176 Nm]

The question asks what is the moment you have to apply to the spindle to keep the disc vertical i.e. to keep the valve shut?

So you need to know the resultant force exerted on the disc by the water and the distance x of this force from the spindle.

We know that the water in the pipe is under a pressure of 3m head of water (to the spindle)

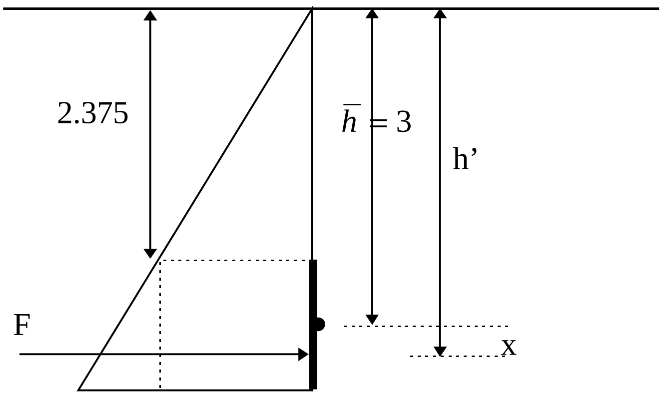


Diagram of the forces on the disc valve, based on an imaginary water surface.

$\bar{h} = 3m$, the depth to the centroid of the disc

h' = depth to the centre of pressure (or line of action of the force)

Calculate the force:

$$F = \rho g \bar{h} A$$
$$= 1000 \times 9.81 \times 3 \times \pi \left(\frac{1.25}{2}\right)^2$$
$$= 36.116\, kN$$

Calculate the line of action of the force, *h'*.

$$h' = \frac{\text{2nd moment of area about water surface}}{\text{1st moment of area about water surface}}$$
$$= \frac{I_{oo}}{A\bar{h}}$$

By the parallel axis theorem 2nd moment of area about O (in the surface) $I_{oo} = I_{GG} + A\bar{h}^2$ where I_{GG} is the 2nd moment of area about a line through the centroid of the disc and $I_{GG} = \pi r^4/4$.

$$h' = \frac{I_{GG}}{A\bar{h}} + \bar{h}$$
$$= \frac{\pi r^4}{4(\pi r^2)3} + 3$$
$$= \frac{r^2}{12} + 3 = 3.0326m$$

So the distance from the spindle to the line of action of the force is

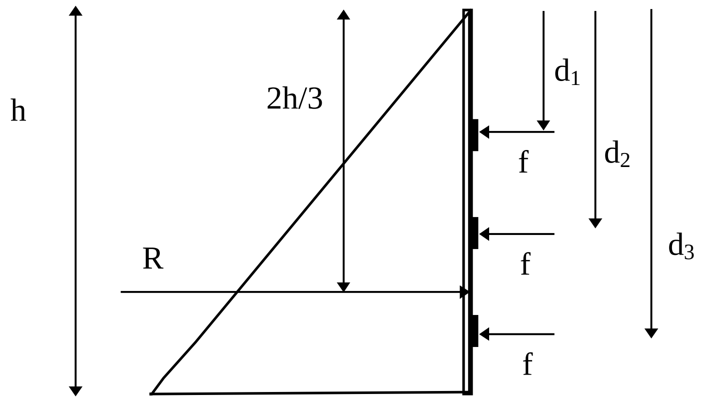
$$x = h' - \bar{h} = 3.0326 - 3 = 0.0326m$$

And the moment required to keep the gate shut is

$$moment = Fx = 36.116 \times 0.0326 = 1.176\,kN\ m$$

2.2
A dock gate is to be reinforced with three horizontal beams. If the water acts on one side only, to a depth of 6m, find the positions of the beams measured from the water surface so that each will carry an equal load. Give the load per meter.
[58 860 N/m, 2.31m, 4.22m, 5.47m]

First of all draw the pressure diagram, as below:



The resultant force per unit length of gate is the area of the pressure diagram. So the total resultant force is

$$R = \frac{1}{2}\rho g h^2 = 0.5 \times 1000 \times 9.81 \times 6^2 = 176580\ N\,(per\ m\ length)$$

Alternatively the resultant force is, R = Pressure at centroid $\times$ Area , (take width of gate as 1m to give force per m)

$$R = \rho g \frac{h}{2} \times (\mathrm{h} \times 1) = 176580\ N\,(per\ m\ length)$$

This is the resultant force exerted by the gate on the water.

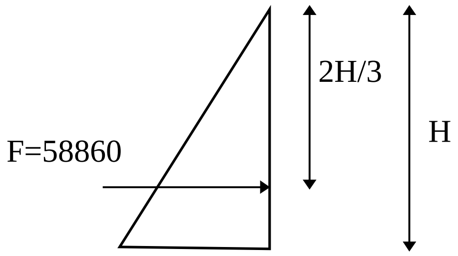
The three beams should carry an equal load, so each beam carries the load *f*, where

$$f = \frac{R}{3} = 58860\ N$$

If we take moments from the surface,

$$DR = fd_1 + fd_2 + fd_3$$
$$D(3f) = f(d_1 + d_2 + d_3)$$
$$12 = d_1 + d_2 + d_3$$

Taking the first beam, we can draw a pressure diagram for this, (ignoring what is below),



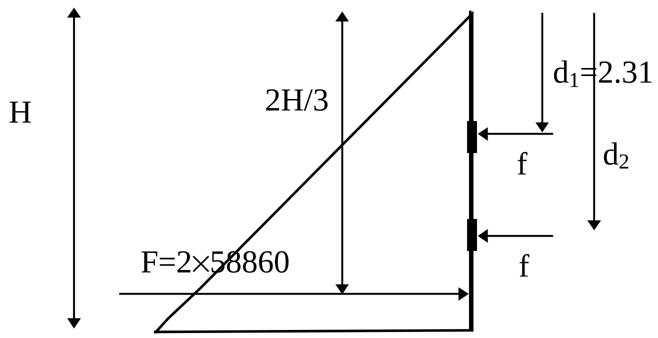
We know that the resultant force, $F = \frac{1}{2}\rho g H^2$, so $H = \sqrt{\frac{2F}{\rho g}}$

$$H = \sqrt{\frac{2F}{\rho g}} = \sqrt{\frac{2 \times 58860}{1000 \times 9.81}} = 3.46\ m$$

And the force acts at 2H/3, so this is the position of the 1st beam,

$$\text{position of 1st beam } = \frac{2}{3}H = 2.31m$$

Taking the second beam into consideration, we can draw the following pressure diagram,



The reaction force is equal to the sum of the forces on each beam, so as before

$$H = \sqrt{\frac{2F}{\rho g}} = \sqrt{\frac{2 \times (2 \times 58860)}{1000 \times 9.81}} = 4.9\ m$$

The reaction force acts at 2H/3, so H=3.27m. Taking moments from the surface,

$$(2 \times 58860) \times 3.27 = 58860 \times 2.31 + 58860 \times d_2$$
$$\text{depth to second beam } \quad d_2 = 4.22\ m$$

For the third beam, from before we have,

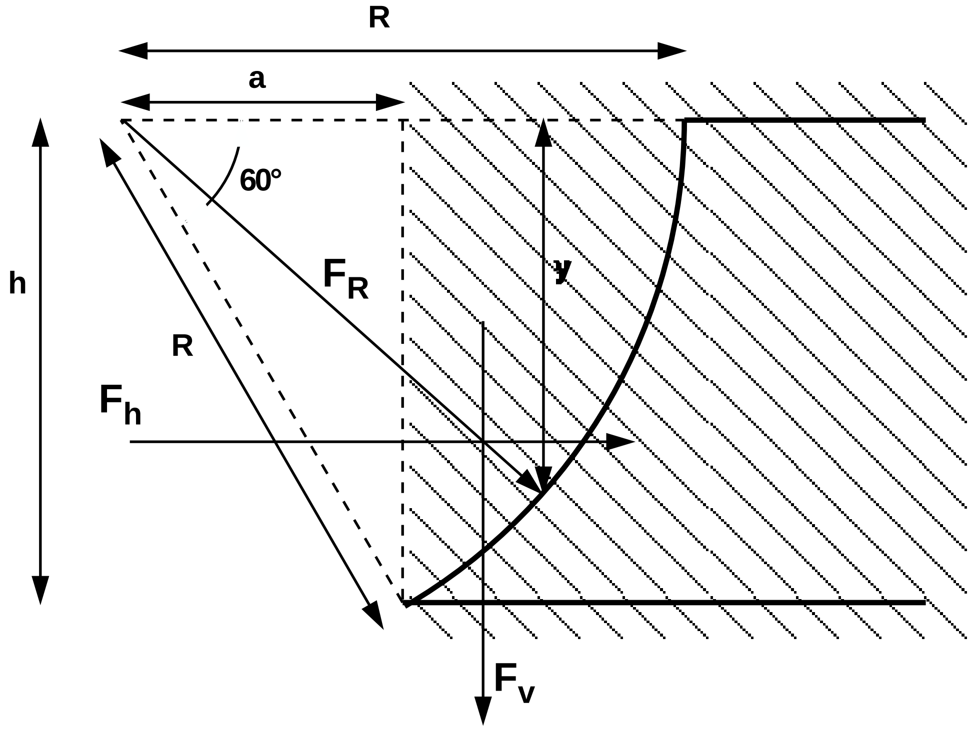
$$12 = d_1 + d_2 + d_3$$
$$\text{depth to third beam } d_3 = 12 - 2.31 - 4.22 = 5.47m$$

2.3

The profile of a masonry dam is an arc of a circle, the arc having a radius of 30m and subtending an angle of 60° at the centre of curvature which lies in the water surface. Determine (a) the load on the dam in N/m length, (b) the position of the line of action to this pressure.

[4.28×10^6 N/m length at depth 19.0m]

Draw the dam to help picture the geometry,



$$h = 30\sin 60 = 25.98 m$$
$$a = 30\cos 60 = 15.0 m$$

Calculate F_v = total weight of fluid above the curved surface (per m length)

$$F_v = \rho g(\text{area of sector - area of triangle})$$
$$= 1000 \times 9.81 \times \left[\left(\pi 30^2 \times \frac{60}{360}\right) - \left(\frac{25.98 \times 15}{2}\right)\right]$$
$$= 2711.375\ kN/m$$

Calculate F_h = force on projection of curved surface onto a vertical plane

$$F_h = \frac{1}{2}\rho g h^2$$
$$= 0.5 \times 1000 \times 9.81 \times 25.98^2 = 3310.681\ kN/m$$

The resultant,

$$F_R = \sqrt{F_v^2 + F_h^2} = \sqrt{3310.681^2 + 2711.375^2}$$
$$= 4279.27\ kN/m$$

acting at the angle

$$\tan\theta = \frac{F_v}{F_h} = 0.819$$

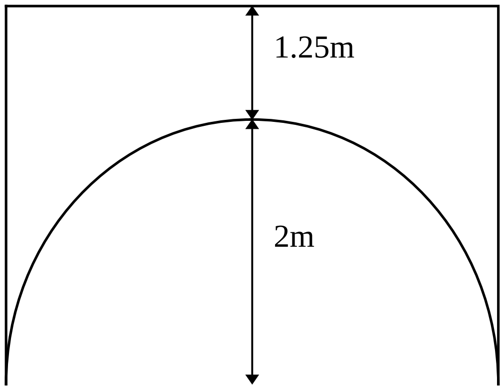
$$\theta = 39.32^\circ$$

As this force act normal to the surface, it must act through the centre of radius of the dam wall. So the depth to the point where the force acts is,

$$y = 30\sin 39.31° = 19\text{m}$$

2.4
The arch of a bridge over a stream is in the form of a semi-circle of radius 2m. the bridge width is 4m. Due to a flood the water level is now 1.25m above the crest of the arch. Calculate (a) the upward force on the underside of the arch, (b) the horizontal thrust on one half of the arch.
[263.6 kN, 176.6 kN]

The bridge and water level can be drawn as:



a) The upward force on the arch = weight of (imaginary) water above the arch.

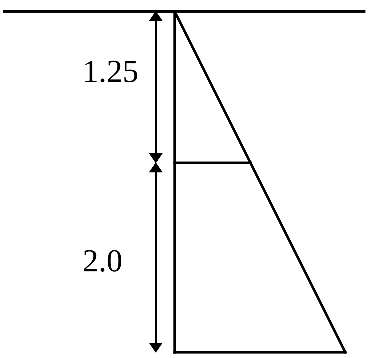
$$R_v = \rho g \times \text{volume of water}$$

$$\text{volume} = \left((1.25+2)\times 4 - \frac{\pi 2^2}{2} \right)\times 4 = 26.867\ m^3$$

$$R_v = 1000\times 9.81\times 26.867 = 263.568\ kN$$

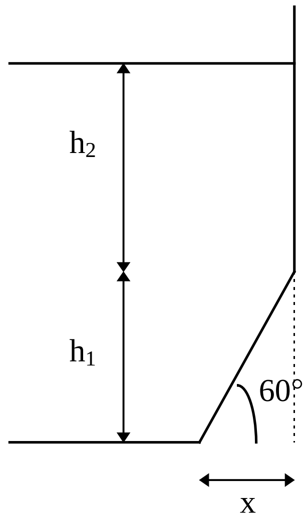
b)

The horizontal force on half of the arch, is equal to the force on the projection of the curved surface onto a vertical plane.



$$\begin{aligned} F_h &= \text{pressure at centroid} \times \text{area} \\ &= \rho g(1.25+1)\times(2\times 4) \\ &= 176.58\ kN \end{aligned}$$

2.5
The face of a dam is vertical to a depth of 7.5m below the water surface then slopes at 30° to the vertical. If the depth of water is 17m what is the resultant force per metre acting on the whole face?
[1563.29 kN]



h_2 = 17.0 m, so h_1 = 17.0 - 7.5 = 9.5 . x = 9.5/tan 60 = 5.485 m.

Vertical force = weight of water above the surface,

$$\begin{aligned} F_v &= \rho g\left(h_2 \times x + 0.5 h_1 \times x\right) \\ &= 9810 \times (7.5 \times 5.485 + 0.5 \times 9.5 \times 5.485) \\ &= 659.123\ kN/m \end{aligned}$$

The horizontal force = force on the projection of the surface on to a vertical plane.

$$\begin{aligned} F_h &= \frac{1}{2}\rho g h^2 \\ &= 0.5 \times 1000 \times 9.81 \times 17^2 \\ &= 1417.545\ kN/m \end{aligned}$$

The resultant force is

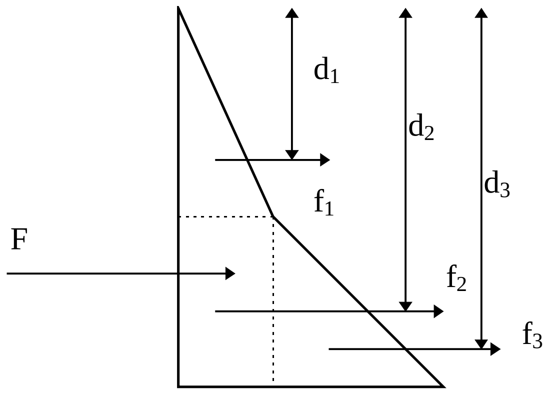
$$\begin{aligned} F_R &= \sqrt{F_v^2 + F_h^2} = \sqrt{659.123^2 + 1417.545^2} \\ &= 1563.29\ kN/m \end{aligned}$$

And acts at the angle

$$\begin{aligned} \tan\theta &= \frac{F_v}{F_h} = 0.465 \\ \theta &= 24.94^{\circ} \end{aligned}$$

2.6
A tank with vertical sides is square in plan with 3m long sides. The tank contains oil of relative density 0.9 to a depth of 2.0m which is floating on water a depth of 1.5m. Calculate the force on the walls and the height of the centre of pressure from the bottom of the tank.
[165.54 kN, 1.15m]

Consider one wall of the tank. Draw the pressure diagram:



density of oil $\rho_{oil} = 0.9\rho_{water} = 900\ kg/m^3$.

Force per unit length, F = area under the graph = sum of the three areas = $f_1 + f_2 + f_3$

$$f_1 = \frac{(900 \times 9.81 \times 2) \times 2}{2} \times 3 = 52974\ N$$
$$f_2 = (900 \times 9.81 \times 2) \times 1.5 \times 3 = 79461\,N$$
$$f_3 = \frac{(1000 \times 9.81 \times 1.5) \times 1.5}{2} \times 3 = 33109\ N$$
$$F = f_1 + f_2 + f_3 = 165544\ N$$

To find the position of the resultant force F, we take moments from any point. We will take moments about the surface.

$$DF = f_1 d_1 + f_2 d_2 + f_3 d_3$$
$$165544\,D = 52974 \times \frac{2}{3}2 + 79461 \times (2 + \frac{1.5}{2}) + 33109 \times (2 + \frac{2}{3}1.5)$$
$$D = 2.347m \text{ (from surface)}$$
$$= 1.153m \text{ (from base of wall)}$$

Application of the Bernoulli Equation

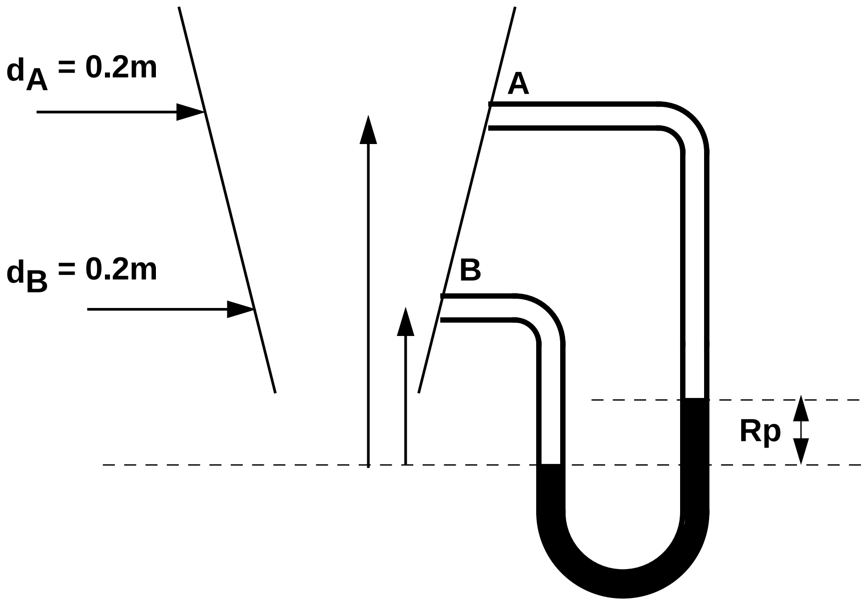
3.1

In a vertical pipe carrying water, pressure gauges are inserted at points A and B where the pipe diameters are 0.15m and 0.075m respectively. The point B is 2.5m below A and when the flow rate down the pipe is 0.02 cumecs, the pressure at B is 14715 N/m^2 greater than that at A.

Assuming the losses in the pipe between A and B can be expressed as $k\dfrac{v^2}{2g}$ where *v* is the velocity at A, find the value of *k*.

If the gauges at A and B are replaced by tubes filled with water and connected to a U-tube containing mercury of relative density 13.6, give a sketch showing how the levels in the two limbs of the U-tube differ and calculate the value of this difference in metres.

[*k* = 0.319, 0.0794m]



Part i)

$$d_A = 0.15m \qquad d_B = 0.075m \qquad Q = 0.02\ m^3/s$$

$$p_B - p_A = 14715\,N/m^2$$

$$h_f = \frac{kv^2}{2g}$$

Taking the datum at B, the Bernoulli equation becomes:

$$\frac{p_A}{\rho g} + \frac{u_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{u_B^2}{2g} + z_B + k\frac{u_A^2}{2g}$$

$$z_A = 2.5 \qquad z_B = 0$$

By continuity: $Q = u_A A_A = u_B A_B$

$$u_A = 0.02/\left(\pi 0.075^2\right) = 1.132\,m/s$$

$$u_B = 0.02/\left(\pi 0.0375^2\right) = 4.527\,m/s$$

giving

$$\frac{p_B - p_A}{1000g} - z_A + \frac{u_B^2 - u_A^2}{2g} = -k\frac{u_A^2}{2g}$$

$$1.5 - 2.5 + 1.045 - 0.065 = -0.065k$$

$$k = 0.319$$

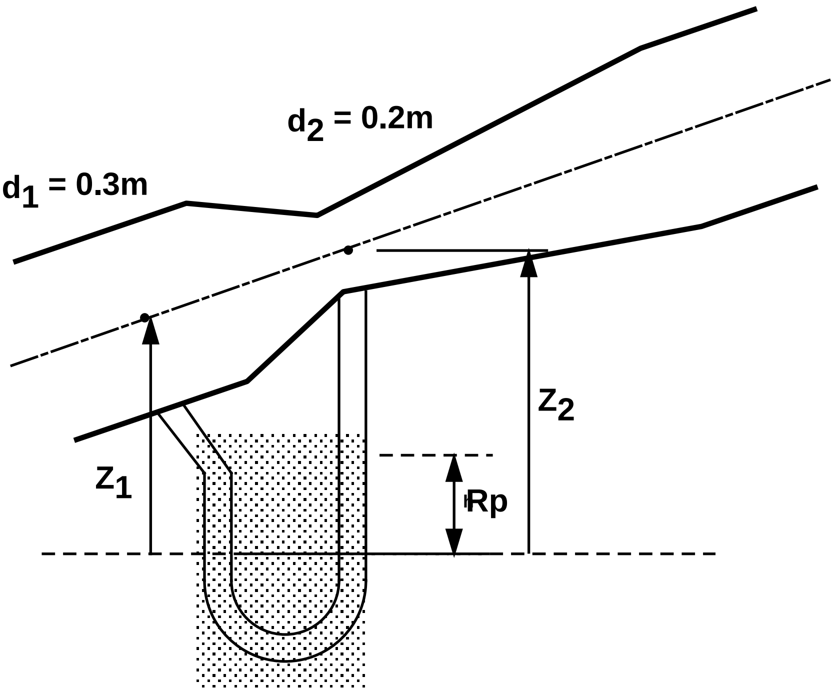
Part ii)

$$p_{xxL} = \rho_w g z_B + p_B$$
$$p_{xxR} = \rho_m g R_p + \rho_w g z_A - \rho_w g R_p + p_A$$

$$p_{xxL} = p_{xxR}$$
$$\rho_w g z_B + p_B = \rho_m g R_p + \rho_w g z_A - \rho_w g R_p + p_A$$
$$p_B - p_A = \rho_w g\left(z_A - z_B\right) + gR_P\left(\rho_m - \rho_w\right)$$
$$14715 = 1000 \times 9.81 \times 2.5 + 9.81 R_p (13600 - 1000)$$
$$R_p = -0.079\, m$$

3.2

A Venturimeter with an entrance diameter of 0.3m and a throat diameter of 0.2m is used to measure the volume of gas flowing through a pipe. The discharge coefficient of the meter is 0.96.
Assuming the specific weight of the gas to be constant at 19.62 N/m^3, calculate the volume flowing when the pressure difference between the entrance and the throat is measured as 0.06m on a water U-tube manometer.
[0.816 m^3/s]



What we know from the question:

$$\rho_g g = 19.62\ N/m^2$$
$$C_d = 0.96$$
$$d_1 = 0.3m$$
$$d_2 = 0.2m$$

Calculate Q.

$$u_1 = Q/0.0707 \qquad\qquad u_2 = Q/0.0314$$

For the manometer:

$$p_1 + \rho_g gz = p_2 + \rho_g g\left(z_2 - R_p\right) + \rho_w gR_p$$
$$p_1 - p_2 = 19.62\left(z_2 - z_1\right) + 587.423 \qquad < - - - - (1)$$

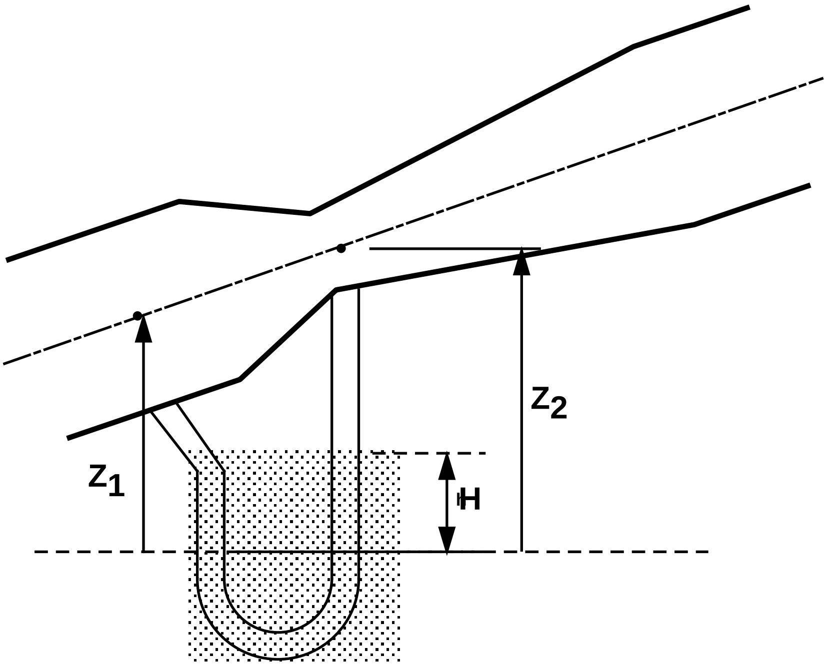
For the Venturimeter

$$\frac{p_1}{\rho_g g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho_g g} + \frac{u_2^2}{2g} + z_2$$
$$p_1 - p_2 = 19.62\left(z_2 - z_1\right) + 0.803u_2^2 \qquad < - - - - (2)$$

Combining (1) and (2)

$$0.803u_2^2 = 587.423$$
$$u_{2\,ideal} = 27.047\ m/s$$
$$Q_{ideal} = 27.047 \times \pi\left(\frac{0.2}{2}\right)^2 = 0.85m^3/s$$
$$Q = C_d Q_{idea} = 0.96 \times 0.85 = 0.816m^3/s$$

3.3
A Venturimeter is used for measuring flow of water along a pipe. The diameter of the Venturi throat is two fifths the diameter of the pipe. The inlet and throat are connected by water filled tubes to a mercury U-tube manometer. The velocity of flow along the pipe is found to be $2.5\sqrt{H}$ m/s, where H is the manometer reading in metres of mercury. Determine the loss of head between inlet and throat of the Venturi when H is 0.49m. (Relative density of mercury is 13.6).
[0.23m of water]



For the manometer:

$$p_1 + \rho_w g z_1 = p_2 + \rho_w g (z_2 - H) + \rho_m g H$$

$$p_1 - p_2 = \rho_w g z_2 - \rho_w g H + \rho_m g H - \rho_w g z_1 \qquad <----(1)$$

For the Venturimeter

$$\frac{p_1}{\rho_w g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho_w g} + \frac{u_2^2}{2g} + z_2 + Losses$$

$$p_1 - p_2 = \frac{\rho_w u_2^2}{2} + \rho_w g z_2 - \frac{\rho_w u_1^2}{2} - \rho_w g z_1 + L \rho_w g \qquad <----(2)$$

Combining (1) and (2)

$$\frac{p_1}{\rho_w g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho_w g} + \frac{u_2^2}{2g} + z_2 + Losses$$

$$L \rho_w g = Hg(\rho_m - \rho_w) - \frac{\rho_w}{2}\left(u_2^2 - u_1^2\right) \qquad <----(3)$$

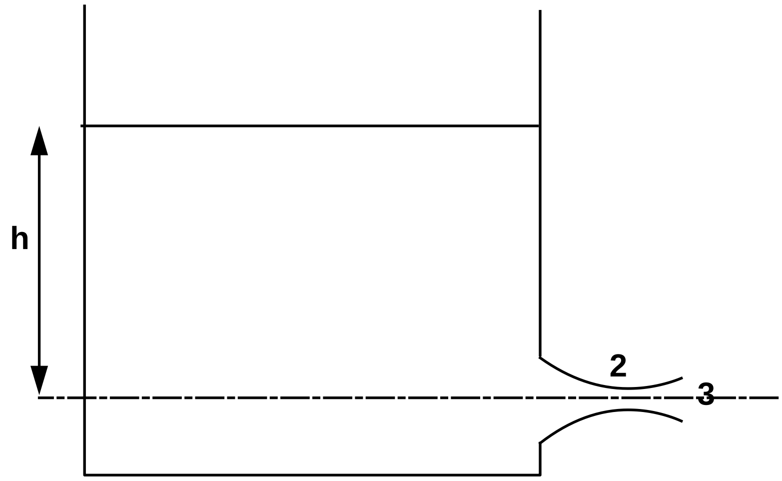
but at 1. From the question

$$u_1 = 2.5\sqrt{H} = 1.75 m/s$$
$$u_1 A_1 = u_2 A_2$$
$$1.75 \times \pi \frac{d^2}{4} = u_2 \pi \left(\frac{2d}{10}\right)^2$$
$$u_2 = 10.937\ m/s$$

Substitute in (3)

$$Losses = L = \frac{0.49 \times 9.81(13600 - 1000) - (1000/2)(10.937^2 - 1.75^2)}{9.81 \times 1000}$$
$$= 0.233m$$

3.4
Water is discharging from a tank through a convergent-divergent mouthpiece. The exit from the tank is rounded so that losses there may be neglected and the minimum diameter is 0.05m.
If the head in the tank above the centre-line of the mouthpiece is 1.83m. a) What is the discharge?
b) What must be the diameter at the exit if the absolute pressure at the minimum area is to be 2.44m of water? c) What would the discharge be if the divergent part of the mouth piece were removed. (Assume atmospheric pressure is 10m of water).
[0.0752m, 0.0266m^3/s, 0.0118m^3/s]



From the question:

$$d_2 = 0.05m$$
$$\text{minimum pressure} = \frac{p_2}{\rho g} = 2.44m$$
$$\frac{p_1}{\rho g} = 10m = \frac{p_3}{\rho g}$$

Apply Bernoulli:

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2 = \frac{p_3}{\rho g} + \frac{u_3^2}{2g} + z_3$$

If we take the datum through the orifice:

$$z_1 = 1.83m \qquad z_2 = z_3 = 0 \qquad u_1 = \text{negligible}$$

Between 1 and 2

$$10 + 1.83 = 2.44 + \frac{u_2^2}{2g}$$
$$u_2 = 13.57\, m/s$$
$$Q = u_2 A_2 = 13.57 \times \pi \left(\frac{0.05}{2}\right)^2 = 0.02665\, m^3/s$$

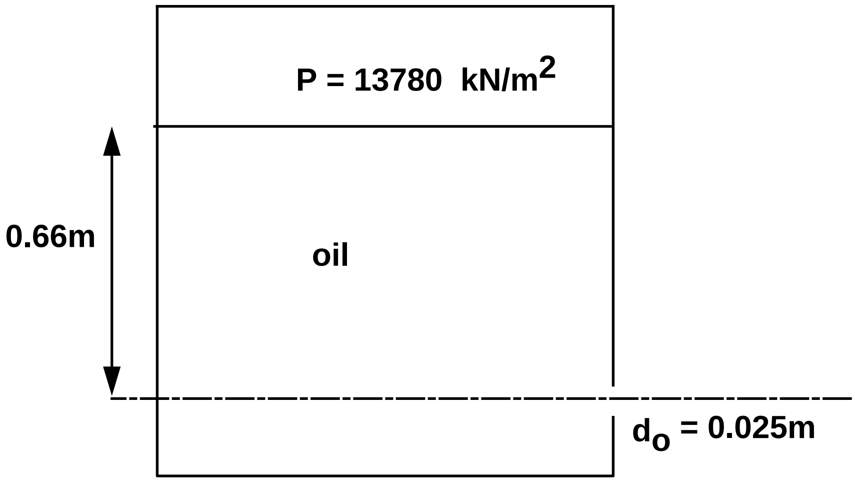
Between 1 and 3 $p_1 = p_3$

$$1.83 = \frac{u_3^2}{2g}$$
$$u_3 = 5.99\, m/s$$
$$Q = u_3 A_3$$
$$0.02665 = 5.99 \times \pi \frac{d_3^2}{4}$$
$$d_3 = 0.0752m$$

If the mouth piece has been removed, $p_1 = p_2$

$$\frac{p_1}{\rho g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g}$$
$$u_2 = \sqrt{2gz_1} = 5.99\, m/s$$
$$Q = 5.99\pi \frac{0.05^2}{4} = 0.0118\, m^3/s$$

3.5
A closed tank has an orifice 0.025m diameter in one of its vertical sides. The tank contains oil to a depth of 0.61m above the centre of the orifice and the pressure in the air space above the oil is maintained at 13780 N/m^2 above atmospheric. Determine the discharge from the orifice.
(Coefficient of discharge of the orifice is 0.61, relative density of oil is 0.9).
[0.00195 m^3/s]



From the question

$$\sigma = 0.9 = \frac{\rho_o}{\rho_w}$$
$$\rho_o = 900$$
$$C_d = 0.61$$

Apply Bernoulli,

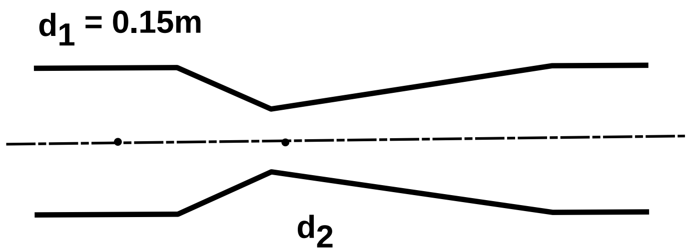
$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

Take atmospheric pressure as 0,

$$\frac{13780}{\rho_o g} + 0.61 = \frac{u_2^2}{2g}$$
$$u_2 = 6.53 m/s$$
$$Q = 0.61 \times 6.53 \times \pi \left(\frac{0.025}{2}\right)^2 = 0.00195 m^3/s$$

3.6
The discharge coefficient of a Venturimeter was found to be constant for rates of flow exceeding a certain value. Show that for this condition the loss of head due to friction in the convergent parts of the meter can be expressed as KQ^2 *m* where *K* is a constant and *Q* is the rate of flow in cumecs.
Obtain the value of *K* if the inlet and throat diameter of the Venturimeter are 0.102m and 0.05m respectively and the discharge coefficient is 0.96.
[*K*=1060]

3.7
A Venturimeter is to fitted in a horizontal pipe of 0.15m diameter to measure a flow of water which may be anything up to 240m^3/hour. The pressure head at the inlet for this flow is 18m above atmospheric and the pressure head at the throat must not be lower than 7m below atmospheric. Between the inlet and the throat there is an estimated frictional loss of 10% of the difference in pressure head between these points. Calculate the minimum allowable diameter for the throat.
[0.063m]



From the question:

$$d_1 = 0.15m \qquad Q = 240 m^3/hr = 0.667 m^3/s$$
$$u_1 = Q/A = 3.77 m/s$$
$$\frac{p_1}{\rho g} = 18m \qquad \frac{p_2}{\rho g} = -7m$$

Friction loss, from the question:

$$h_f = 0.1\frac{(p_1 - p_2)}{\rho g}$$

Apply Bernoulli:

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + h_f$$

$$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} + \frac{u_1^2}{2g} - h_f = \frac{u_2^2}{2g}$$

$$25 - \frac{3.77^2}{2g} - 2.5 = \frac{u_2^2}{2g}$$

$$u_2 = 21.346\, m/s$$

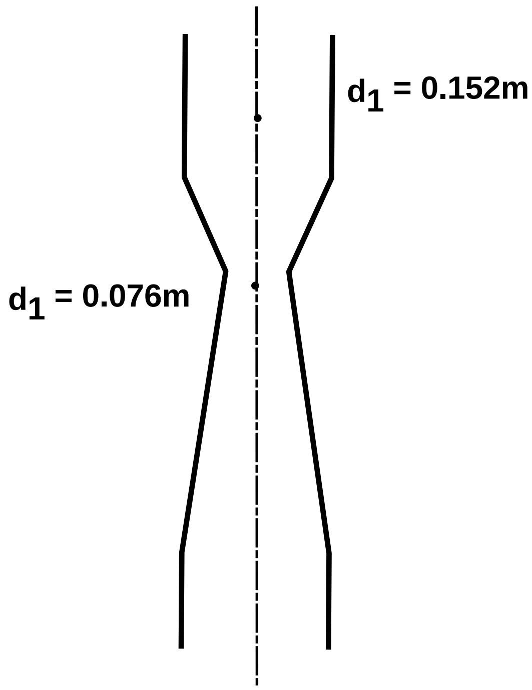
$$Q = u_2 A_2$$

$$0.0667 = 21.346 \times \pi \frac{d_2^2}{4}$$

$$d_2 = 0.063m$$

3.8

A Venturimeter of throat diameter 0.076m is fitted in a 0.152m diameter vertical pipe in which liquid of relative density 0.8 flows downwards. Pressure gauges are fitted to the inlet and to the throat sections. The throat being 0.914m below the inlet. Taking the coefficient of the meter as 0.97 find the discharge
a) when the pressure gauges read the same b)when the inlet gauge reads 15170 N/m^2 higher than the throat gauge.
[0.0192m^3/s, 0.034m^3/s]



From the question:

$$d_1 = 0.152m \qquad A_1 = 0.01814m$$
$$d_2 = 0.076m \qquad A_2 = 0.00454m$$
$$\rho = 800\,kg/m^3$$
$$C_d = 0.97$$

Apply Bernoulli:

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

a) $p_1 = p_2$

$$\frac{u_1^2}{2g} + z_1 = \frac{u_2^2}{2g} + z_2$$

By continuity:

$$Q = u_1 A_1 = u_2 A_2$$
$$u_2 = u_1 \frac{A_1}{A_2} = u_1 4$$

$$\frac{u_1^2}{2g} + 0.914 = \frac{16u_1^2}{2g}$$
$$u_1 = \sqrt{\frac{0.914 \times 2 \times 9.81}{15}} = 1.0934\,m/s$$
$$Q = C_d A_1 u_1$$
$$Q = 0.96 \times 0.01814 \times 1.0934 = 0.019\,m^3/s$$

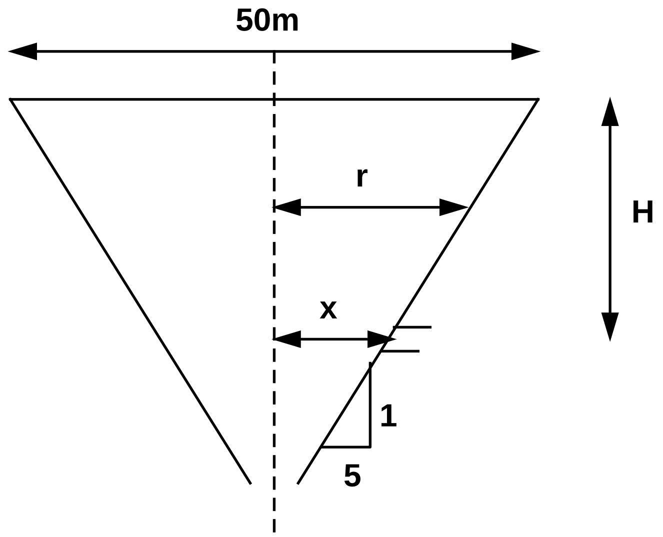
b)

$$p_1 - p_2 = 15170$$
$$\frac{p_1 - p_2}{\rho g} = \frac{u_2^2 - u_1^2}{2g} - 0.914$$
$$\frac{15170}{\rho g} = \frac{Q^2\left(220.43^2 - 55.11^2\right)}{2g} - 0.914$$
$$55.8577 = Q^2\left(220.43^2 - 55.11^2\right)$$
$$Q = 0.035\,m^3/s$$

Tank emptying

4.1

A reservoir is circular in plan and the sides slope at an angle of $\tan^{-1}(1/5)$ to the horizontal. When the reservoir is full the diameter of the water surface is 50m. Discharge from the reservoir takes place through a pipe of diameter 0.65m, the outlet being 4m below top water level. Determine the time for the water level to fall 2m assuming the discharge to be $0.75a\sqrt{2gH}$ cumecs where a is the cross sectional area of the pipe in m^2 and H is the head of water above the outlet in m.
[1325 seconds]



From the question: H = 4m a = $\pi(0.65/2)^2 = 0.33\text{m}^2$

$$Q = 0.75a\sqrt{2gh}$$
$$= 1.0963\sqrt{h}$$

In time δt the level in the reservoir falls δh, so

$$Q\,\delta t = -A\,\delta h$$
$$\delta t = -\frac{A}{Q}\delta h$$

Integrating give the total time for levels to fall from h_1 to h_2.

$$T = -\int_{h_1}^{h_2} \frac{A}{Q}\,dh$$

As the surface area changes with height, we must express A in terms of h.

$$A = \pi r^2$$

But r varies with h.

It varies linearly from the surface at $H = 4m$, $r = 25m$, at a gradient of $\tan^{-1} = 1/5$.

$$r = x + 5h$$

$$25 = x + 5(4)$$

$$x = 5$$

so $A = \pi(5 + 5h)^2 = (25\pi + 25\pi h^2 + 50\pi h)$

Substituting in the integral equation gives

$$T = -\int_{h_1}^{h_2} \frac{25\pi + 25\pi h^2 + 50\pi h}{1.0963\sqrt{h}}\, dh$$

$$= -\frac{25\pi}{1.0963}\int_{h_1}^{h_2} \frac{1 + h^2 + 2h}{\sqrt{h}}\, dh$$

$$= -71.641\int_{h_1}^{h_2} \frac{1}{\sqrt{h}} + \frac{h^2}{\sqrt{h}} + \frac{2h}{\sqrt{h}}\, dh$$

$$= -71.641\int_{h_1}^{h_2} h^{-1/2} + h^{3/2} + 2h^{1/2}\, dh$$

$$= -71.641\left[2h^{1/2} + \frac{2}{5}h^{53/2} + \frac{4}{3}h^{3/2}\right]_{h_1}^{h_2}$$

From the question, $h_1 = 4m$ $\quad$ $h_2 = 2m$, *so*

$$T = -71.641\left[\left(2\times 4^{1/2} + \frac{2}{5}\times 4^{53/2} + \frac{4}{3}\times 4^{3/2}\right) - \left(2\times 2^{1/2} + \frac{2}{5}\times 2^{53/2} + \frac{4}{3}\times 2^{3/2}\right)\right]$$

$$= -71.641\left[(4 + 12.8 + 10.667) - (2.828 + 2.263 + 3.77)\right]$$

$$= -71.641[27.467 - 8.862]$$

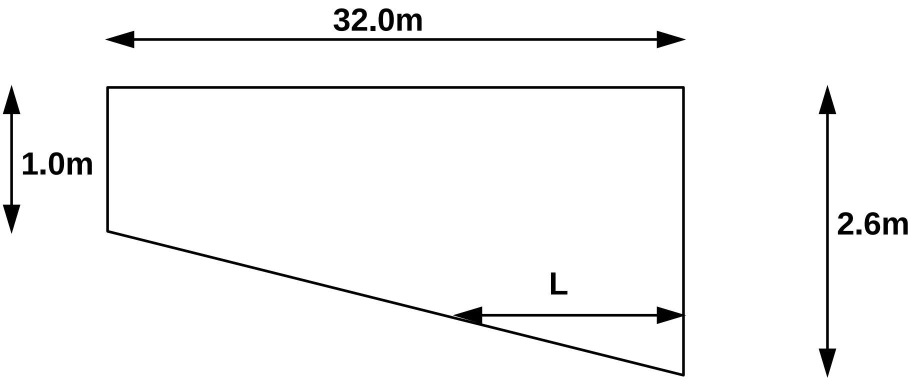
$$= 1333 \text{ sec}$$

4.2

A rectangular swimming pool is 1m deep at one end and increases uniformly in depth to 2.6m at the other end. The pool is 8m wide and 32m long and is emptied through an orifice of area 0.224m^2, at the lowest point in the side of the deep end. Taking C_d for the orifice as 0.6, find, from first principles,

a) the time for the depth to fall by 1m b) the time to empty the pool completely.

[299 second, 662 seconds]



The question tell us $a_o = 0.224m^2$, $C_d = 0.6$

Apply Bernoulli from the tank surface to the vena contracta at the orifice:

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

$p_1 = p_2$ and $u_1 = 0$. $u_2 = \sqrt{2gh}$

We need Q in terms of the height h measured above the orifice.

$$Q = C_d a_o u_2 = C_d a_o \sqrt{2gh}$$
$$= 0.6 \times 0.224 \times \sqrt{2 \times 9.81}\sqrt{h}$$
$$= 0.595\sqrt{h}$$

And we can write an equation for the discharge in terms of the surface height change:

$$Q\,\delta t = -A\,\delta h$$
$$\delta t = -\frac{A}{Q}\delta h$$

Integrating give the total time for levels to fall from h_1 to h_2.

$$T = -\int_{h_1}^{h_2} \frac{A}{Q}\,dh$$
$$= -1.68\int_{h_1}^{h_2} \frac{A}{\sqrt{h}}\,dh \qquad <------- (1)$$

a) For the first 1m depth, A = 8 x 32 = 256, whatever the *h*.

So, for the first period of time:

$$T = -1.68\int_{h_1}^{h_2} \frac{256}{\sqrt{h}}\,dh$$
$$= -430.08\left[\sqrt{h_1} - \sqrt{h_2}\right]$$
$$= -430.08\left[\sqrt{2.6} - \sqrt{1.6}\right]$$
$$= 299\ \text{sec}$$

b) now we need to find out how long it will take to empty the rest.

We need the area *A*, in terms of *h*.

$$A = 8L$$
$$\frac{L}{h} = \frac{32}{1.6}$$
$$A = 160h$$

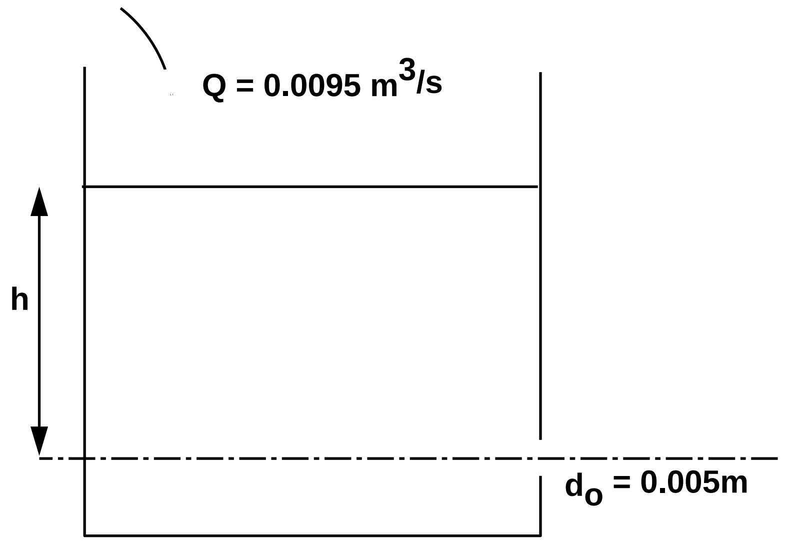
So

$$T = -1.68\int_{h_1}^{h_2} \frac{160h}{\sqrt{h}}\,dh$$
$$= -268.9\,\frac{2}{3}\left[(h_1)^{3/2} - (h_2)^{3/2}\right]$$
$$= -268.9\,\frac{2}{3}\left[(1.6)^{3/2} - (0)^{3/2}\right]$$
$$= 362.67\ \text{sec}$$

Total time for emptying is,

T = 363 + 299 = 662 sec

4.3
A vertical cylindrical tank 2m diameter has, at the bottom, a 0.05m diameter sharp edged orifice for which the discharge coefficient is 0.6.
a) If water enters the tank at a constant rate of 0.0095 cumecs find the depth of water above the orifice when the level in the tank becomes stable.
b) Find the time for the level to fall from 3m to 1m above the orifice when the inflow is turned off.
c) If water now runs into the tank at 0.02 cumecs, the orifice remaining open, find the rate of rise in water level when the level has reached a depth of 1.7m above the orifice.
[a) 3.314m, b) 881 seconds, c) 0.252m/min]



From the question: $Q_{in} = 0.0095\ m^3/s$, $d_o=0.05m$, $C_d = 0.6$

Apply Bernoulli from the water surface (1) to the orifice (2),

$$\frac{p_1}{\rho g}+\frac{u_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{u_2^2}{2g}+z_2$$

$p_1 = p_2$ and $u_1 = 0$. $u_2 = \sqrt{2gh}$.

With the datum the bottom of the cylinder, $z_1 = h$, $z_2 = 0$

We need Q in terms of the height h measured above the orifice.

$$\begin{aligned}Q_{out} &= C_d a_o u_2 = C_d a_o \sqrt{2gh}\\ &= 0.6\pi\left(\frac{0.05}{2}\right)^2\sqrt{2\times 9.81}\sqrt{h}\\ &= 0.00522\sqrt{h} \qquad <-----(1)\end{aligned}$$

For the level in the tank to remain constant:

inflow = out flow

$$Q_{in} = Q_{out}$$

$$0.0095 = 0.00522\sqrt{h}$$

$$h = 3.314m$$

(b) Write the equation for the discharge in terms of the surface height change:

$$Q\,\delta t = -A\,\delta h$$

$$\delta t = -\frac{A}{Q}\delta h$$

Integrating between h_1 and h_2, to give the time to change surface level

$$T = -\int_{h_1}^{h_2} \frac{A}{Q}\,dh$$

$$= -601.8\int_{h_1}^{h_2} h^{-1/2}\ dh$$

$$= -1203.6\left[h^{1/2}\right]_{h_1}^{h_2}$$

$$= -1203.6\left[h_2^{1/2} - h_1^{1/2}\right]$$

$h_1 = 3$ and $h_2 = 1$ so

$$T = 881\ sec$$

c) Q_{in} changed to $Q_{in} = 0.02\ m^3/s$

From (1) we have $Q_{out} = 0.00522\sqrt{h}$. The question asks for the rate of surface rise when $h = 1.7m$.

i.e. $$Q_{out} = 0.00522\sqrt{1.7} = 0.0068\,m^3/s$$

The rate of increase in volume is:

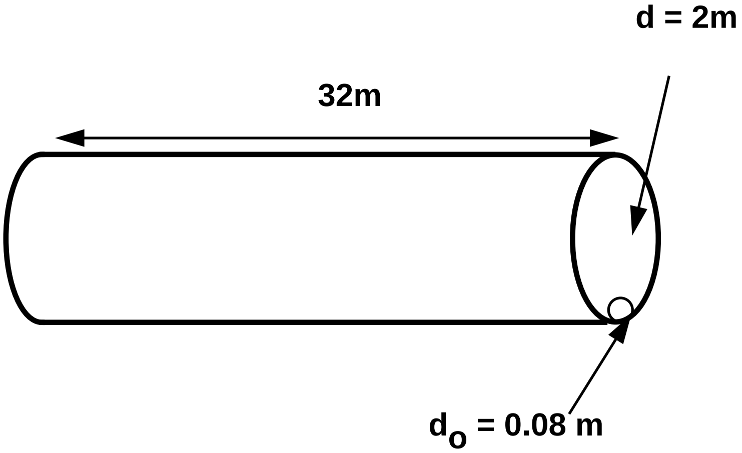
$$Q = Q_{in} - Q_{out} = 0.02 - 0.0068 = 0.0132\,m^3/s$$

As $Q = Area\ x\ Velocity$, the rate of rise in surface is

$$Q = Au$$

$$u = \frac{Q}{A} = \frac{0.0132}{\left(\frac{\pi 2^2}{4}\right)} = 0.0042\,m/s = 0.252\,m/\min$$

4.4
A horizontal boiler shell (i.e. a horizontal cylinder) 2m diameter and 10m long is half full of water. Find the time of emptying the shell through a short vertical pipe, diameter 0.08m, attached to the bottom of the shell. Take the coefficient of discharge to be 0.8.
[1370 seconds]



From the question $W = 10m$, $D = 10m$ $d_o = 0.08m$ $C_d = 0.8$

Apply Bernoulli from the water surface (1) to the orifice (2),

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

$p_1 = p_2$ and $u_1 = 0$. $u_2 = \sqrt{2gh}$.

With the datum the bottom of the cylinder, $z_1 = h$, $z_2 = 0$

We need Q in terms of the height h measured above the orifice.

$$\begin{aligned} Q_{out} &= C_d a_o u_2 = C_d a_o \sqrt{2gh} \\ &= 0.8\pi\left(\frac{0.08}{2}\right)^2 \sqrt{2 \times 9.81}\sqrt{h} \\ &= 0.0178\sqrt{h} \end{aligned}$$

Write the equation for the discharge in terms of the surface height change:

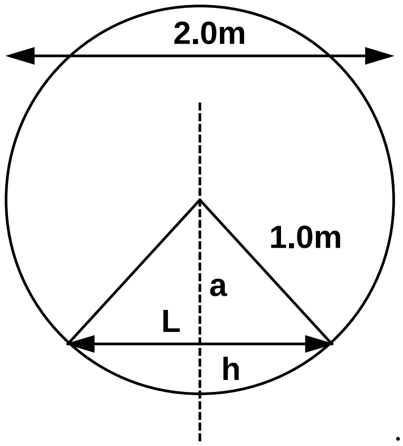
$$Q\,\delta t = -A\,\delta h$$

$$\delta t = -\frac{A}{Q}\delta h$$

Integrating between h_1 and h_2, to give the time to change surface level

$$T = -\int_{h_1}^{h_2} \frac{A}{Q}\,dh$$

But we need A in terms of h



Surface area $A = 10L$, so need L in terms of h

$$1^2 = a^2 + \left(\frac{L}{2}\right)^2$$

$$a = (1-h)$$

$$1^2 = (1-h)^2 + \left(\frac{L}{2}\right)^2$$

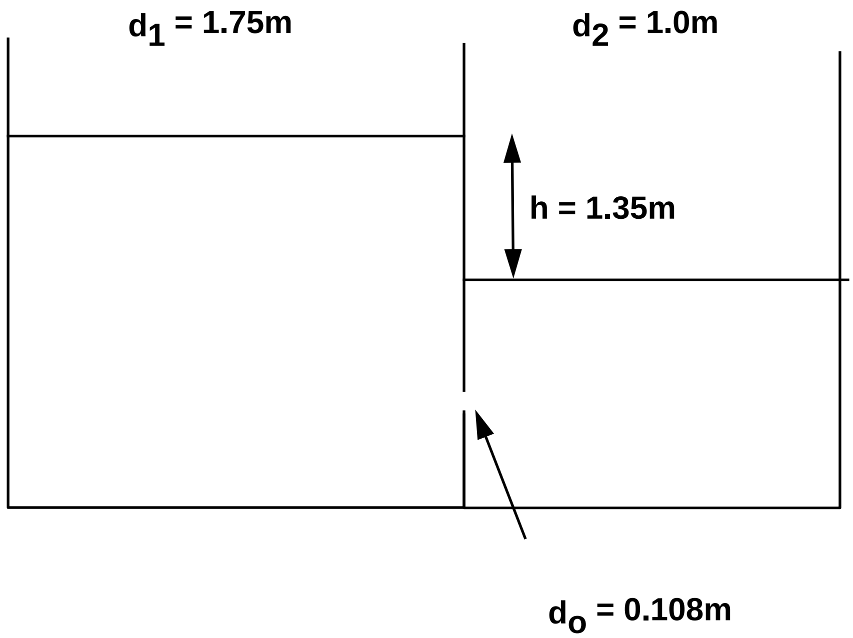
$$L = 2\sqrt{\left(2h-h^2\right)}$$

$$A = 20\sqrt{2h-h^2}$$

Substitute this into the integral term,

$$T = -\int_{h_1}^{h_2} \frac{20\sqrt{2h-h^2}}{0.1078\sqrt{h}}\,dh$$
$$= -1123.6\int_{h_1}^{h_2} \frac{\sqrt{2h-h^2}}{\sqrt{h}}\,dh$$
$$= -1123.6\int_{h_1}^{h_2} \sqrt{\frac{2h-h^2}{h}}dh$$
$$= -1123.6\int_{h_1}^{h_2} \sqrt{2-h}\,dh$$
$$= -1123.6\left(-\frac{2}{3}\right)\left[(2-h)^{3/2}\right]_{h_1}^{h_2}$$
$$= 749.07[2.828-1] = 1369.6\,\text{sec}$$

4.5
Two cylinders standing upright contain liquid and are connected by a submerged orifice. The diameters of the cylinders are 1.75m and 1.0m and of the orifice, 0.08m. The difference in levels of the liquid is initially 1.35m. Find how long it will take for this difference to be reduced to 0.66m if the coefficient of discharge for the orifice is 0.605. (Work from first principles.)
[30.7 seconds]



$$A_1 = \pi\left(\frac{1.75}{2}\right)^2 = 2.4m^2 \qquad A_2 = \pi\left(\frac{1}{2}\right)^2 = 0.785m^2$$

$$d_o = 0.08m, \quad a_o = \pi\left(\frac{0.08}{2}\right)^2 = 0.00503m^2 \qquad C_d = 0.605$$

by continuity,

$$-A_1\delta h_1 = A_2\delta h_2 = Q\delta t \qquad <----(1)$$

defining, $h = h_1 - h_2$

$$-\delta h = -\delta h_1 + \delta h_2$$

Substituting this in (1) to eliminate δh_2

$$-A_1\delta h_1 = A_2(\delta h_1 - \delta h) = A_2\delta h_1 - A_2\delta h$$

$$\delta h_1 = \frac{A_2\delta h}{A_1 + A_2}$$

$$-A_1\frac{A_2\delta h}{A_1 + A_2} = Q\delta t \qquad <-----(2)$$

From the Bernoulli equation we can derive this expression for discharge through the submerged orifice:

$$Q = C_d a_o\sqrt{2gh}$$

So

$$-A_1\frac{A_2\delta h}{A_1 + A_2} = C_d a_o\sqrt{2gh}\ \delta t$$

$$\delta t = -\frac{A_1 A_2}{\left(A_1 + A_2\right)C_d a_o\sqrt{2g}}\frac{1}{\sqrt{h}}\delta h$$

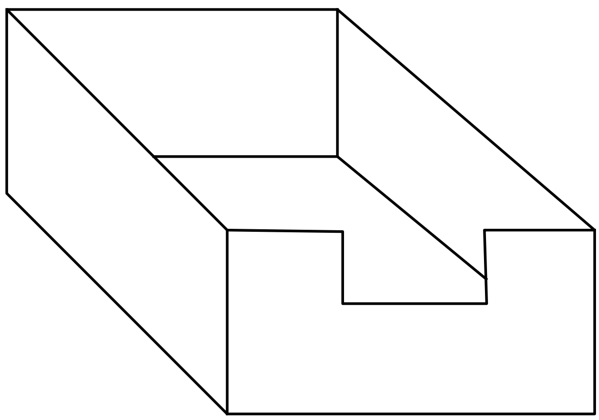
Integrating

$$\begin{aligned}
T &= -\frac{A_1 A_2}{\left(A_1 + A_2\right)C_d a_o\sqrt{2g}}\int_{h_1}^{h_2}\frac{1}{\sqrt{h}}dh \\
&= -\frac{2A_1 A_2}{\left(A_1 + A_2\right)C_d a_o\sqrt{2g}}\left(\sqrt{h_2} - \sqrt{h_1}\right) \\
&= \frac{-2\times 2.4\times 0.785}{(2.4+0.785)\times 0.605\times 0.00503\sqrt{2\times 9.81}}(0.8124 - 1.1619) \\
&= 30.7\sec
\end{aligned}$$

4.6

A rectangular reservoir with vertical walls has a plan area of 60000m^2. Discharge from the reservoir take place over a rectangular weir. The flow characteristics of the weir is $Q = 0.678\,H^{3/2}$ cumecs where H is the depth of water above the weir crest. The sill of the weir is 3.4m above the bottom of the reservoir. Starting with a depth of water of 4m in the reservoir and no inflow, what will be the depth of water after one hour?

[3.98m]



From the question $A = 60\,000\ m^2$, $Q = 0.678\ h^{\ 3/2}$

Write the equation for the discharge in terms of the surface height change:

$$Q\,\delta t = -A\,\delta h$$

$$\delta t = -\frac{A}{Q}\delta h$$

Integrating between h_1 and h_2, to give the time to change surface level

$$T = -\int_{h_1}^{h_2} \frac{A}{Q}\,dh$$
$$= -\frac{60000}{0.678}\int_{h_1}^{h_2} \frac{1}{h^{3/2}}\,dh$$
$$= 2\times 88495.58\left[h^{-1/2}\right]_{h_1}^{h_2}$$

From the question $T = 3600\ sec$ and $h_1 = 0.6m$

$$3600 = 176991.15\left[h_2^{-1/2} - 0.6^{-1/2}\right]$$
$$h_2 = 0.5815m$$

Total depth = 3.4 + 0.58 = 3.98m

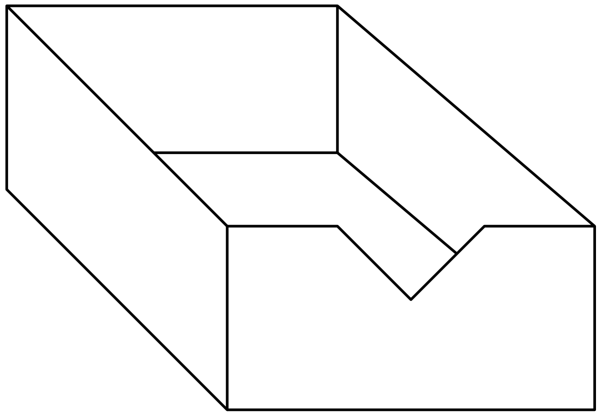
Notches and weirs

5.1

Deduce an expression for the discharge of water over a right-angled sharp edged V-notch, given that the coefficient of discharge is 0.61.

A rectangular tank 16m by 6m has the same notch in one of its short vertical sides. Determine the time taken for the head, measured from the bottom of the notch, to fall from 15cm to 7.5cm.

[1399 seconds]



From your notes you can derive:

$$Q = \frac{8}{15} C_d \tan\frac{\theta}{2} \sqrt{2g} H^{5/2}$$

For this weir the equation simplifies to

$$Q = 1.44 H^{5/2}$$

Write the equation for the discharge in terms of the surface height change:

$$Q\,\delta t = -A\,\delta h$$

$$\delta t = -\frac{A}{Q}\delta h$$

Integrating between h_1 and h_2, to give the time to change surface level

$$\begin{aligned} T &= -\int_{h_1}^{h_2} \frac{A}{Q}\,dh \\ &= -\frac{16\times 6}{1.44}\int_{h_1}^{h_2} \frac{1}{h^{5/2}}\,dh \\ &= \frac{2}{3}\times 66.67\left[h^{-3/2}\right]_{h_1}^{h_2} \end{aligned}$$

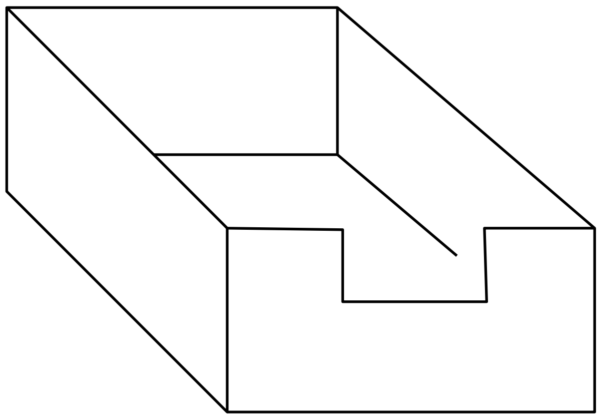
$h_1 = 0.15m$, $h_2 = 0.075m$

$$\begin{aligned} T &= 44.44\left[0.075^{-3/2} - 0.15^{-3/2}\right] \\ &= 1399\,\text{sec} \end{aligned}$$

5.2

Derive an expression for the discharge over a sharp crested rectangular weir. A sharp edged weir is to be constructed across a stream in which the normal flow is 200 litres/sec. If the maximum flow likely to occur in the stream is 5 times the normal flow then determine the length of weir necessary to limit the rise in water level to 38.4cm above that for normal flow. $C_d=0.61$.

[1.24m]



From your notes you can derive:

$$Q = \frac{2}{3} C_d b \sqrt{2g} h^{3/2}$$

From the question:

$$Q_1 = 0.2\ m^3/s, \qquad h_1 = x$$

$$Q_2 = 1.0\ m^3/s, \qquad h_2 = x + 0.384$$

where x is the height above the weir at normal flow.

So we have two situations:

$$0.2 = \frac{2}{3} C_d b \sqrt{2g} x^{3/2} = 1.801 b x^{3/2} \qquad <--(1)$$

$$1.0 = \frac{2}{3} C_d b \sqrt{2g} (x + 0.384)^{3/2} = 1.801 b (x + 0.384)^{3/2} \qquad <--(2)$$

From (1) we get an expression for b in terms of x

$$b = 0.111 x^{-3/2}$$

Substituting this in (2) gives,

$$1.0 = 1.801 \times 0.111 \left(\frac{x + 0.384}{x} \right)^{3/2}$$

$$5^{2/3} = \left(\frac{x + 0.384}{x} \right)$$

$$x = 0.1996 m$$

So the weir breadth is

$$\begin{aligned} b &= 0.111 (0.1996)^{-3/2} \\ &= 1.24 m \end{aligned}$$

5.3
Show that the rate of flow across a triangular notch is given by $Q=C_dKH^{5/2}$ cumecs, where C_d is an experimental coefficient, K depends on the angle of the notch, and H is the height of the undisturbed water level above the bottom of the notch in metres. State the reasons for the introduction of the coefficient.
Water from a tank having a surface area of 10m^2 flows over a 90° notch. It is found that the time taken to lower the level from 8cm to 7cm above the bottom of the notch is 43.5seconds. Determine the coefficient C_d assuming that it remains constant during his period.
[0.635]

The proof for $Q = \frac{8}{15} C_d \tan\frac{\theta}{2}\sqrt{2g}H^{5/2} = C_d KH^{5/2}$ is in the notes.

From the question:

$$A = 10m^2 \qquad \theta = 90^\circ \qquad h_1 = 0.08m \qquad h_2 = 0.07m \qquad T = 43.5sec$$

So

$$Q = 2.36\, C_d\, h^{5/2}$$

Write the equation for the discharge in terms of the surface height change:

$$Q\,\delta t = -A\,\delta h$$

$$\delta t = -\frac{A}{Q}\delta h$$

Integrating between h_1 and h_2, to give the time to change surface level

$$T = -\int_{h_1}^{h_2} \frac{A}{Q}\, dh$$

$$= -\frac{10}{2.36C_d}\int_{h_1}^{h_2} \frac{1}{h^{5/2}}\, dh$$

$$= \frac{2}{3}\times\frac{4.23}{C_d}\left[h^{-3/2}\right]_{0.07}^{0.08}$$

$$43.5 = \frac{2.82}{C_d}\left[0.07^{-3/2} - 0.08^{-3/2}\right]$$

$$C_d = 0.635$$

5.4
A reservoir with vertical sides has a plan area of 56000m^2. Discharge from the reservoir takes place over a rectangular weir, the flow characteristic of which is $Q=1.77BH^{3/2}$ m^3/s. At times of maximum rainfall, water flows into the reservoir at the rate of 9m^3/s. Find a) the length of weir required to discharge this quantity if head must not exceed 0.6m; b) the time necessary for the head to drop from 60cm to 30cm if the inflow suddenly stops.
[10.94m, 3093seconds]

From the question:

$$A = 56000\ m^2 \qquad Q = 1.77\,B\,H^{3/2} \qquad Q_{max} = 9\ m^3/s$$

a) Find B for H = 0.6

$$9 = 1.77\,B\,0.6^{3/2}$$

$$B = 10.94m$$

b) Write the equation for the discharge in terms of the surface height change:

$$Q\,\delta t = -A\,\delta h$$

$$\delta t = -\frac{A}{Q}\,\delta h$$

Integrating between h_1 and h_2, to give the time to change surface level

$$\begin{aligned} T &= -\int_{h_1}^{h_2} \frac{A}{Q}\,dh \\ &= -\frac{56000}{1.77B}\int_{h_1}^{h_2} \frac{1}{h^{3/2}}\,dh \\ &= \frac{2\times 56000}{1.77B}\left[h^{-1/2}\right]_{0.6}^{0.3} \\ &= 5784\left[0.3^{-1/2} - 0.6^{-1/2}\right] \\ T &= 3093\,\text{sec} \end{aligned}$$

5.5
Develop a formula for the discharge over a 90° V-notch weir in terms of head above the bottom of the V. A channel conveys 300 litres/sec of water. At the outlet end there is a 90° V-notch weir for which the coefficient of discharge is 0.58. At what distance above the bottom of the channel should the weir be placed in order to make the depth in the channel 1.30m? With the weir in this position what is the depth of water in the channel when the flow is 200 litres/sec?
[0.755m, 1.218m]

Derive this formula from the notes: $Q = \frac{8}{15}C_d \tan\frac{\theta}{2}\sqrt{2g}H^{5/2}$

From the question:

$\theta = 90°$ $\quad C_d\ 0.58$ $\quad Q = 0.3\ m^3/s,$ $\quad$ depth of water, $Z = 0.3m$

giving the weir equation:

$$Q = 1.37H^{5/2}$$

a) As H is the height above the <u>bottom of the V</u>, the depth of water = $Z = D + H$, where D is the height of the bottom of the V from the base of the channel. So

$$\begin{aligned} Q &= 1.37(Z - D)^{5/2} \\ 0.3 &= 1.37(1.3 - D)^{5/2} \\ D &= 0.755m \end{aligned}$$

b) Find Z when $Q = 0.2\ m^3/s$

$$\begin{aligned} 0.2 &= 1.37(Z - 0.755)^{5/2} \\ Z &= 1.218m \end{aligned}$$

5.6

Show that the quantity of water flowing across a triangular V-notch of angle 2θ is $Q = C_d \frac{8}{15} \tan\theta \sqrt{2g} H^{5/2}$. Find the flow if the measured head above the bottom of the V is 38cm, when θ=45° and C_d=0.6. If the flow is wanted within an accuracy of 2%, what are the limiting values of the head.

[0.126m^3/s, 0.377m, 0.383m]

Proof of the v-notch weir equation is in the notes.

From the question:

$H = 0.38m$ $\theta = 45^\circ$ $C_d = 0.6$

The weir equation becomes:

$$\begin{aligned} Q &= 1.417 H^{5/2} \\ &= 1.417(0.38)^{5/2} \\ &= 0.126\ m^3/s \end{aligned}$$

$Q+2\% = 0.129\ m^3/s$

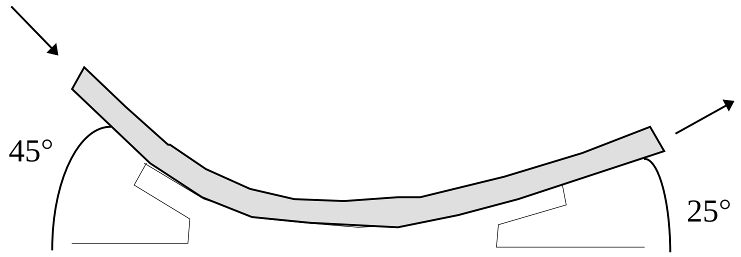
$$\begin{aligned} 0.129 &= 1.417 H^{5/2} \\ H &= 0.383m \end{aligned}$$

$Q-2\% = 0.124\ m^3/s$

$$\begin{aligned} 0.124 &= 1.417 H^{5/2} \\ H &= 0.377m \end{aligned}$$

Application of the Momentum Equation

6.1
The figure below shows a smooth curved vane attached to a rigid foundation. The jet of water, rectangular in section, 75mm wide and 25mm thick, strike the vane with a velocity of 25m/s. Calculate the vertical and horizontal components of the force exerted on the vane and indicate in which direction these components act.
[Horizontal 233.4 N acting from right to left. Vertical 1324.6 N acting downwards]



From the question:

$$a_1 = 0.075 \times 0.025 = 1.875 \times 10^{-3}\ m^2$$
$$u_1 = 25\, m/s$$
$$Q = 1.875 \times 10^{-3} \times 25\, m^3/s$$
$$a_1 = a_2, \qquad so \qquad u_1 = u_2$$

Calculate the total force using the momentum equation:

$$F_{T\,x} = \rho Q\left(u_2 \cos 25 - u_1 \cos 45\right)$$
$$= 1000 \times 0.0469(25 \cos 25 - 25 \cos 45)$$
$$= 233.44\ N$$

$$F_{T\,y} = \rho Q\left(u_2 \sin 25 - u_1 \sin 45\right)$$
$$= 1000 \times 0.0469(25 \sin 25 - 25 \sin 45)$$
$$= 1324.6\ N$$

Body force and pressure force are 0.

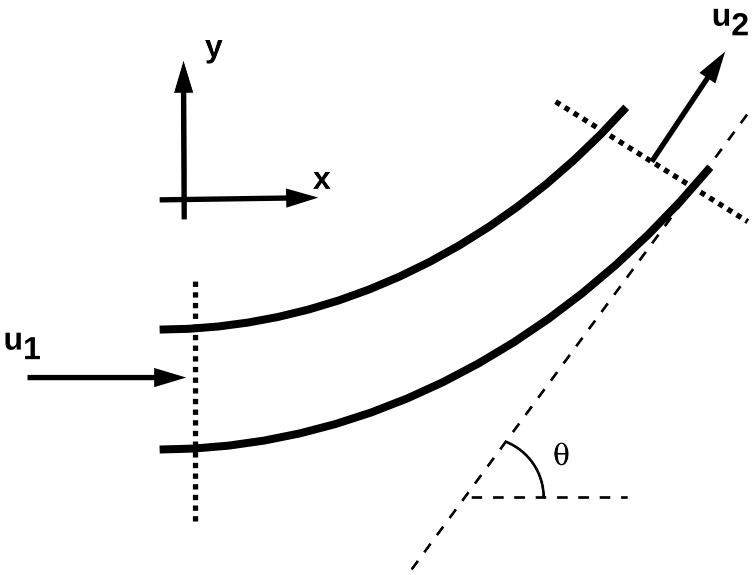
So force on vane:

$$R_x = -F_{t\,x} = -233.44 N$$
$$R_y = -F_{t\,y} = -1324.6 N$$

6.2

A 600mm diameter pipeline carries water under a head of 30m with a velocity of 3m/s. This water main is fitted with a horizontal bend which turns the axis of the pipeline through 75° (i.e. the internal angle at the bend is 105°). Calculate the resultant force on the bend and its angle to the horizontal.

[104.044 kN, 52° 29']



From the question:

$$a = \pi\left(\frac{0.6}{2}\right)^2 = 0.283\,m^2 \qquad d = 0.6\,m \qquad h = 30\,m$$

$$u_1 = u_2 = 3\,m/s \qquad Q = 0.848\,m^3/s$$

Calculate total force.

$$F_{Tx} = \rho Q\left(u_{2x} - u_{1x}\right) = F_{Rx} + F_{Px} + F_{Bx}$$

$$F_{Tx} = 1000 \times 0.848(3\cos 75 - 3) = -1.886\,kN$$

$$F_{Ty} = \rho Q\left(u_{2y} - u_{1y}\right) = F_{Ry} + F_{Py} + F_{By}$$

$$F_{Ty} = 1000 \times 0.848(3\sin 75 - 0) = 2.457\,kN$$

Calculate the pressure force

$$p_1 = p_2 = p = h\rho g = 30 \times 1000 \times 9.81 = 294.3\ kN/m^2$$

$$\begin{aligned} F_{Tx} &= p_1 a_1 \cos\theta_1 - p_2 a_2 \cos\theta_2 \\ &= 294300 \times 0.283(1 - \cos 75) \\ &= 61.73\,kN \end{aligned}$$

$$\begin{aligned} F_{Ty} &= p_1 a_1 \sin\theta_1 - p_2 a_2 \sin\theta_2 \\ &= 294300 \times 0.283(0 - \sin 75) \\ &= -80.376\,kN \end{aligned}$$

There is no body force in the x or y directions.

$$F_{Rx} = F_{Tx} - F_{Px} - F_{Bx}$$
$$= -1.886 - 61.73 - 0 = -63.616\,kN$$

$$F_{Ry} = F_{Ty} - F_{Py} - F_{By}$$
$$= 2.457 + 80.376 - 0 = -82.833\,kN$$

These forces act <u>on the fluid</u>

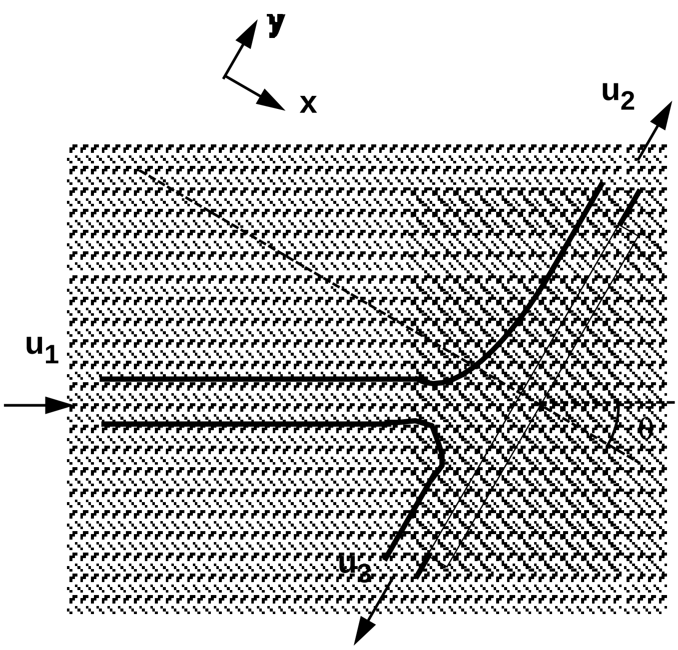
The resultant force on the fluid is

$$F_R = \sqrt{F_{Rx} + F_{Ry}} = 104.44\,kN$$

$$\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = 52^{\circ}29'$$

6.3

A horizontal jet of water 2×10^3 mm^2 cross-section and flowing at a velocity of 15 m/s hits a flat plate at 60° to the axis (of the jet) and to the horizontal. The jet is such that there is no side spread. If the plate is stationary, calculate a) the force exerted on the plate in the direction of the jet and b) the ratio between the quantity of fluid that is deflected upwards and that downwards. (Assume that there is no friction and therefore no shear force.)
[338N, 3:1]



From the question $a_2 = a_3 = 2x10^{-3}\ m^2$ $u = 15\ m/s$

Apply Bernoulli,

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2 = \frac{p_3}{\rho g} + \frac{u_3^2}{2g} + z_3$$

Change in height is negligible so $z_1 = z_2 = z_3$ and pressure is always atmospheric $p_1 = p_2 = p_3 = 0$. So

$u_1 = u_2 = u_3 = 15\ m/s$

By continuity $Q_1 = Q_2 + Q_3$

$u_1 a_1 = u_2 a_2 + u_3 a_3$

so $a_1 = a_2 + a_3$

Put the axes normal to the plate, as we know that the resultant force is normal to the plate.

$$Q_1 = a_1 u = 2\times10^{-3}\times15 = 0.03$$

$$Q_1 = (a_2 + a_3)\ u$$

$$Q_2 = a_2 u$$

$$Q_3 = (a_1 - a_2)u$$

Calculate total force.

$$F_{Tx} = \rho Q\left(u_{2x} - u_{1x}\right) = F_{Rx} + F_{Px} + F_{Bx}$$

$$F_{Tx} = 1000\times0.03(0 - 15\sin 60) = 390\ N$$

Component in direction of jet $= 390 \sin 60 = 338\ N$

As there is no force parallel to the plate $F_{ty} = 0$

$$F_{Ty} = \rho u_2^2 a_2 - \rho u_3^2 a_3 - \rho u_1^2 a_1 \cos\theta = 0$$

$$a_2 - a_3 - a_1 \cos\theta = 0$$

$$a_1 = a_2 + a_3$$

$$a_3 + a_1 \cos\theta = a_1 - a_3$$

$$4a_3 = a_1 = \frac{4}{3}a_2$$

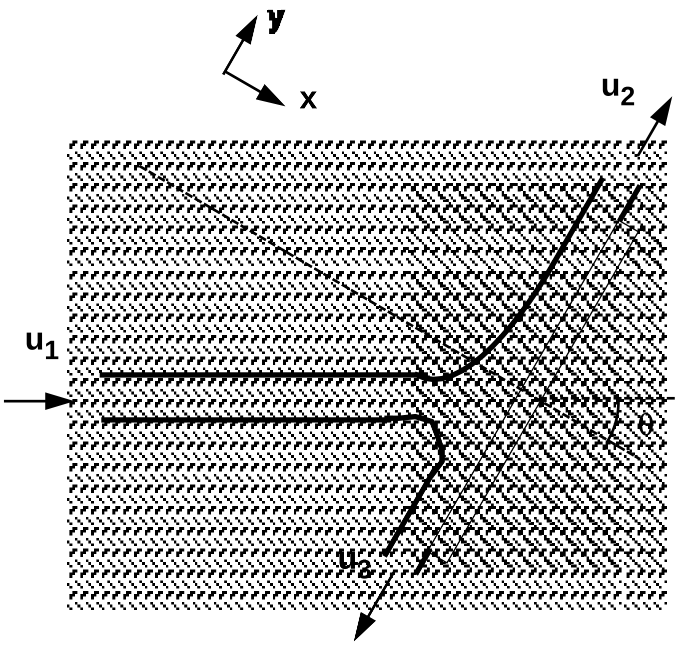
$$a_3 = \frac{1}{3}a_2$$

Thus 3/4 of the jet goes up, 1/4 down

6.4

A 75mm diameter jet of water having a velocity of 25m/s strikes a flat plate, the normal of which is inclined at 30° to the jet. Find the force normal to the surface of the plate.

[2.39kN]



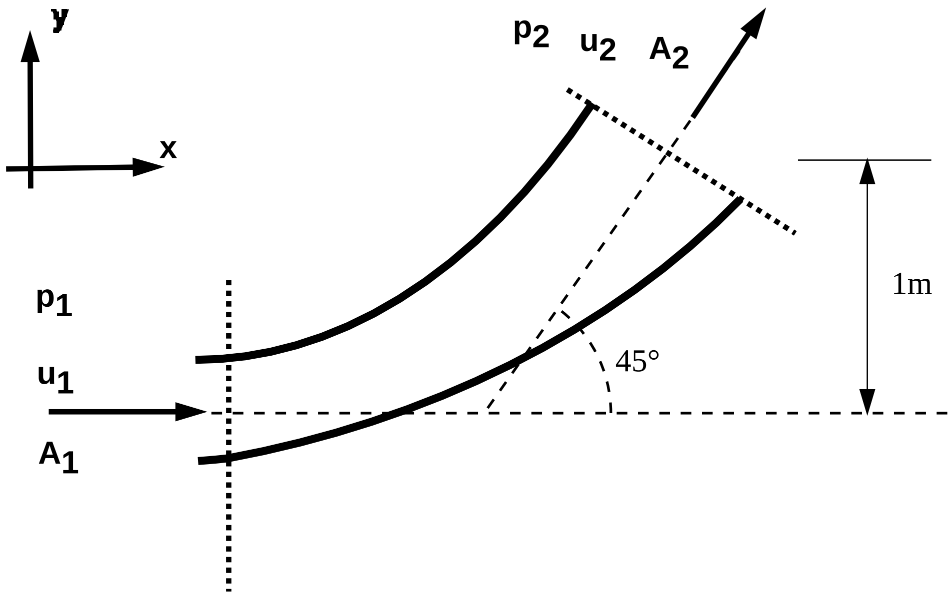
From the question, $d_{jet} = 0.075m$ $u_1=25m/s$ $Q = 25\pi(0.075/2)^2 = 0.11\ m^3/s$

Force normal to plate is

$$F_{Tx} = \rho Q(0 - u_{1x})$$

$$F_{Tx} = 1000\times 0.11 (0 - 25 \cos 30) = 2.39\ kN$$

6.5
The outlet pipe from a pump is a bend of 45° rising in the vertical plane (i.e. and internal angle of 135°). The bend is 150mm diameter at its inlet and 300mm diameter at its outlet. The pipe axis at the inlet is horizontal and at the outlet it is 1m higher. By neglecting friction, calculate the force and its direction if the inlet pressure is 100kN/m² and the flow of water through the pipe is 0.3m³/s. The volume of the pipe is 0.075m³.
[13.94kN at 67° 40' to the horizontal]



1&2 Draw the control volume and the axis system

$p_1 = 100$ kN/m², $Q = 0.3$ m³/s $\theta = 45°$

$d_1 = 0.15$ m $d_2 = 0.3$ m

$A_1 = 0.177$ m² $A_2 = 0.0707$ m²

3 Calculate the total force

in the x direction

$$F_{T\,x} = \rho Q\left(u_{2\,x} - u_{1\,x}\right)$$
$$= \rho Q\left(u_2 \cos\theta - u_1\right)$$

by continuity $A_1 u_1 = A_2 u_2 = Q$, so

$$u_1 = \frac{0.3}{\pi\left(0.15^2 / 4\right)} = 16.98\, m/s$$

$$u_2 = \frac{0.3}{0.0707} = 4.24\, m/s$$

$$\begin{aligned} F_{T\,x} &= 1000 \times 0.3(4.24\cos 45 - 16.98) \\ &= -1493.68\, N \end{aligned}$$

and in the y-direction

$$\begin{aligned} F_{T\,y} &= \rho Q\left(u_{2\,y} - u_{1\,y}\right) \\ &= \rho Q\left(u_2 \sin\theta - 0\right) \\ &= 1000 \times 0.3(4.24 \sin 45) \\ &= 899.44\, N \end{aligned}$$

4 <u>Calculate the pressure force.</u>

$$F_P = \text{pressure force at 1 - pressure force at 2}$$

$$F_{P\,x} = p_1 A_1 \cos 0 - p_2 A_2 \cos\theta = p_1 A_1 - p_2 A_2 \cos\theta$$

$$F_{P\,y} = p_1 A_1 \sin 0 - p_2 A_2 \sin\theta = -p_2 A_2 \sin\theta$$

We know pressure at the inlet but not at the outlet.

we can use Bernoulli to calculate this unknown pressure.

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2 + h_f$$

where h_f is the friction loss

In the question it says this can be ignored, $h_f=0$

The height of the pipe at the outlet is 1m above the inlet.

Taking the inlet level as the datum:

$z_1 = 0$ $z_2 = 1$m

So the Bernoulli equation becomes:

$$\frac{100000}{1000 \times 9.81} + \frac{16.98^2}{2 \times 9.81} + 0 = \frac{p_2}{1000 \times 9.81} + \frac{4.24^2}{2 \times 9.81} + 1.0$$

$$p_2 = 225361.4\ N/m^2$$

$$F_{P_x} = 100000 \times 0.0177 - 225361.4 \cos 45 \times 0.0707$$
$$= 1770 - 11266.34 = -9496.37\, kN$$

$$F_{P_y} = -225361.4 \sin 45 \times 0.0707$$
$$= -11266.37$$

5 Calculate the body force

The only body force is the force due to gravity. That is the weight acting in the y direction.

$$F_{B_y} = -\rho g \times volume$$
$$= -1000 \times 9.81 \times 0.075$$
$$= -12901.56\, N$$

There are no body forces in the x direction,

$$F_{B_x} = 0$$

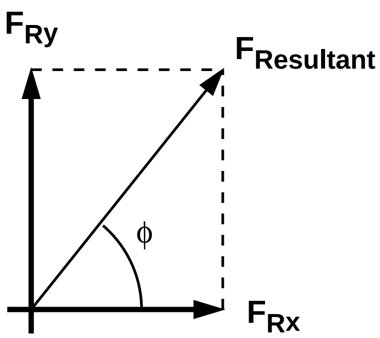
6 Calculate the resultant force

$$F_{T_x} = F_{R_x} + F_{P_x} + F_{B_x}$$
$$F_{T_y} = F_{R_y} + F_{P_y} + F_{B_y}$$

$$F_{R_x} = F_{T_x} - F_{P_x} - F_{B_x}$$
$$= -4193.6 + 9496.37$$
$$= 5302.7\, N$$

$$F_{R_y} = F_{T_y} - F_{P_y} - F_{B_y}$$
$$= 899.44 + 11266.37 + 735.75$$
$$= 12901.56\, N$$

And the resultant force on the fluid is given by



$$F_R = \sqrt{F_{R\,x}^2 - F_{R\,y}^2}$$
$$= \sqrt{5302.7^2 + 12901.56^2}$$
$$= 13.95\,kN$$

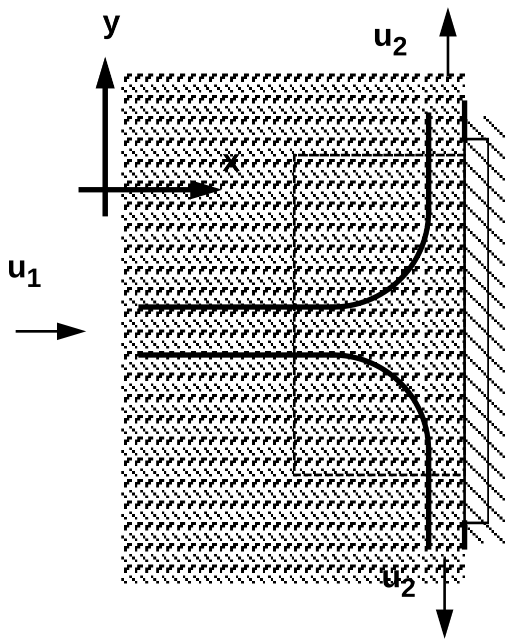
And the direction of application is

$$\phi = \tan^{-1}\left(\frac{F_{R\,y}}{F_{R\,x}}\right) = \tan^{-1}\left(\frac{12901.56}{5302.7}\right) = 67.66^{\circ}$$

The force <u>on the bend</u> is the same magnitude but in the opposite direction

$$R = -F_R$$

6.6
The force exerted by a 25mm diameter jet against a flat plate normal to the axis of the jet is 650N. What is the flow in m^3/s?
[0.018 m^3/s]



From the question, $d_{jet} = 0.025m$ $F_{Tx} = 650\ N$

Force normal to plate is

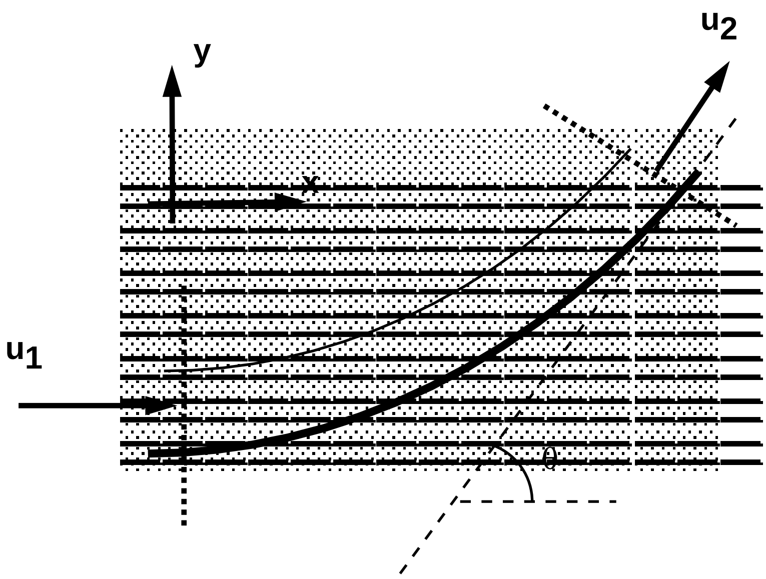
$$F_{Tx} = \rho Q(0 - u_{1x})$$
$$650 = 1000 \times Q\,(0 - u)$$

$Q = au = (\pi d^2/4)u$

$$650 = -1000au^2 = -1000Q^2/a$$
$$650 = -1000Q^2/(\pi 0.025^2/4)$$
$$Q = 0.018m^3/s$$

6.7

A curved plate deflects a 75mm diameter jet through an angle of 45°. For a velocity in the jet of 40m/s to the right, compute the components of the force developed against the curved plate. (Assume no friction).
[R_x=2070N, R_y=5000N down]



From the question:

$$a_1 = \pi 0.075^2 / 4 = 4.42 \times 10^{-3}\, m^2$$
$$u_1 = 40\, m/s$$
$$Q = 4.42 \times 10^{-3} \times 40 = 0.1767\, m^3 / s$$
$$a_1 = a_2, \qquad so \qquad u_1 = u_2$$

Calculate the total force using the momentum equation:

$$\begin{aligned} F_{T\,x} &= \rho Q(u_2 \cos 45 - u_1) \\ &= 1000 \times 0.1767(40 \cos 45 - 40) \\ &= -2070.17\, N \end{aligned}$$

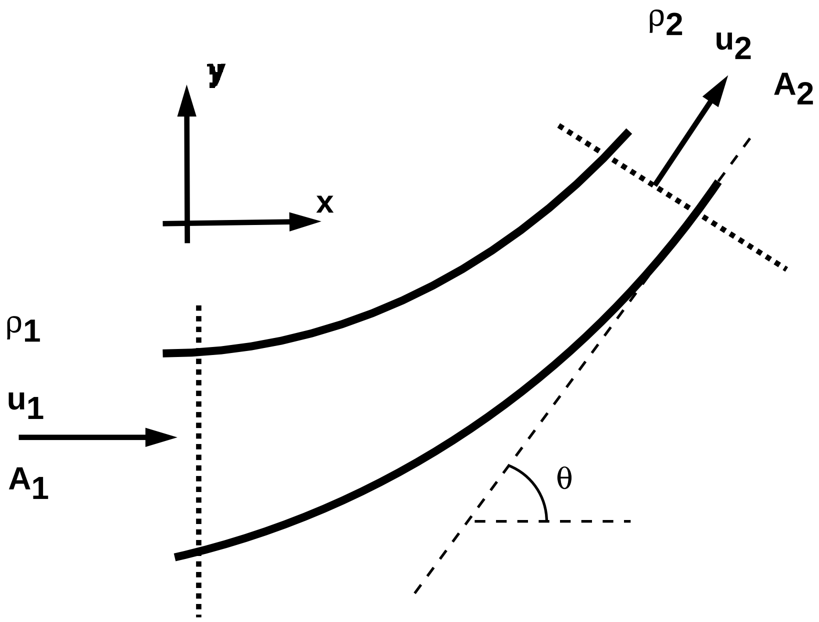
$$\begin{aligned} F_{T\,y} &= \rho Q(u_2 \sin 45 - 0) \\ &= 1000 \times 0.1767(40 \sin 45) \\ &= 4998\, N \end{aligned}$$

Body force and pressure force are 0.

So force on vane:

$$R_x = -F_{t\,x} = 2070 N$$
$$R_y = -F_{t\,y} = -4998 N$$

6.8
A 45° reducing bend, 0.6m diameter upstream, 0.3m diameter downstream, has water flowing through it at the rate of 0.45m^3/s under a pressure of 1.45 bar. Neglecting any loss is head for friction, calculate the force exerted by the water on the bend, and its direction of application.
[R=34400N to the right and down, θ = 14°]



1&2 Draw the control volume and the axis system

$p_1 = 1.45\times10^5$ N/m^2,	Q = 0.45 m^3/s	θ = 45°
$d_1 = 0.6$ m	$d_2 = 0.3$ m	
$A_1 = 0.283$ m^2	$A_2 = 0.0707$ m^2	

3 Calculate the total force

in the x direction

$$F_{T\,x} = \rho Q\left(u_{2\,x} - u_{1\,x}\right)$$
$$= \rho Q\left(u_2 \cos\theta - u_1\right)$$

by continuity $A_1 u_1 = A_2 u_2 = Q$, so

$$u_1 = \frac{0.45}{\pi\left(0.6^2 / 4\right)} = 1.59\, m/s$$
$$u_2 = \frac{0.45}{0.0707} = 6.365\, m/s$$

$$F_{T\,x} = 1000 \times 0.45(6.365\cos 45 - 1.59)$$
$$= 1310\,N$$

and in the y-direction

$$F_{T\,y} = \rho Q\left(u_{2\,y} - u_{1\,y}\right)$$
$$= \rho Q(u_2 \sin\theta - 0)$$
$$= 1000 \times 0.45(6.365 \sin 45)$$
$$= 1800\,N$$

4 Calculate the pressure force.

F_P = pressure force at 1 - pressure force at 2

$$F_{P\,x} = p_1 A_1 \cos 0 - p_2 A_2 \cos\theta = p_1 A_1 - p_2 A_2 \cos\theta$$

$$F_{P\,y} = p_1 A_1 \sin 0 - p_2 A_2 \sin\theta = -p_2 A_2 \sin\theta$$

We know pressure at the inlet but not at the outlet.

we can use Bernoulli to calculate this unknown pressure.

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2 + h_f$$

where h_f is the friction loss

In the question it says this can be ignored, $h_f=0$

Assume the pipe to be horizontal

$z_1 = z_2$

So the Bernoulli equation becomes:

$$\frac{145000}{1000 \times 9.81} + \frac{1.59^2}{2 \times 9.81} = \frac{p_2}{1000 \times 9.81} + \frac{6.365^2}{2 \times 9.81}$$
$$p_2 = 126007 \;\; N/m^2$$

$$F_{P\,x} = 145000 \times 0.283 - 126000 \cos 45 \times 0.0707$$
$$= 41035 - 6300 = 34735\,N$$

$$F_{P\,y} = -126000 \sin 45 \times 0.0707$$
$$= -6300\,N$$

5 Calculate the body force

The only body force is the force due to gravity.

There are no body forces in the x or y directions,

$$F_{B_x} = F_{B_y} = 0$$

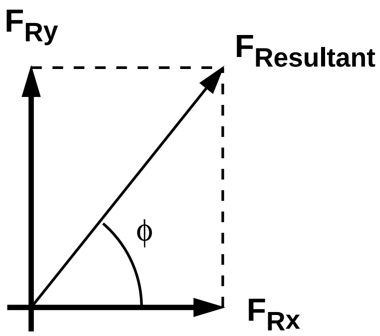
6 Calculate the resultant force

$$F_{T_x} = F_{R_x} + F_{P_x} + F_{B_x}$$
$$F_{T_y} = F_{R_y} + F_{P_y} + F_{B_y}$$

$$\begin{aligned} F_{R_x} &= F_{T_x} - F_{P_x} - F_{B_x} \\ &= 1310 - 34735 \\ &= -33425\, N \end{aligned}$$

$$\begin{aligned} F_{R_y} &= F_{T_y} - F_{P_y} - F_{B_y} \\ &= 1800 + 6300 \\ &= 8100\, N \end{aligned}$$

And the resultant force on the fluid is given by



$$\begin{aligned} F_R &= \sqrt{F_{R_x}^2 - F_{R_y}^2} \\ &= \sqrt{33425^2 + 8100^2} \\ &= 34392\, kN \end{aligned}$$

And the direction of application is

$$\phi = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{8100}{-33425}\right) = 13.62^{\circ}$$

The force on the bend is the same magnitude but in the opposite direction

$$R = -F_R$$

Laminar pipe flow.

7.1
The distribution of velocity, u, in metres/sec with radius r in metres in a smooth bore tube of 0.025 m bore follows the law, $u = 2.5 - kr^2$. Where k is a constant. The flow is laminar and the velocity at the pipe surface is zero. The fluid has a coefficient of viscosity of 0.00027 kg/m s. Determine (a) the rate of flow in m^3/s (b) the shearing force between the fluid and the pipe wall per metre length of pipe.
[6.14×10^{-4} m^3/s, 8.49×10^{-3} N]

The velocity at distance r from the centre is given in the question:

$$u = 2.5 - kr^2$$

Also we know: $\mu = 0.00027$ kg/ms $2r = 0.025$m

We can find k from the boundary conditions:
when $r = 0.0125$, $u = 0.0$ (boundary of the pipe)

$$0.0 = 2.5 - k0.0125^2$$
$$k = 16000$$

$$u = 2.5 - 1600\, r^2$$

a)
Following along similar lines to the derivation seen in the lecture notes, we can calculate the flow δQ through a small annulus δr:

$$\delta Q = u_r A_{annulus}$$
$$A_{annulus} = \pi(r+\delta r)^2 - \pi r^2 \approx 2\pi r\delta r$$
$$\delta Q = \left(2.5 - 16000r^2\right)2\pi r\delta r$$
$$Q = 2\pi \int_0^{0.0125} \left(2.5r - 16000r^3\right)\, dr$$
$$= 2\pi \left[\frac{2.5r^2}{2} - \frac{16000}{4}r^4\right]_0^{0.0125}$$
$$= 6.14\, m^3/s$$

b)

The shear force is given by $F = \tau \times (2\pi r)$

From Newtons law of viscosity

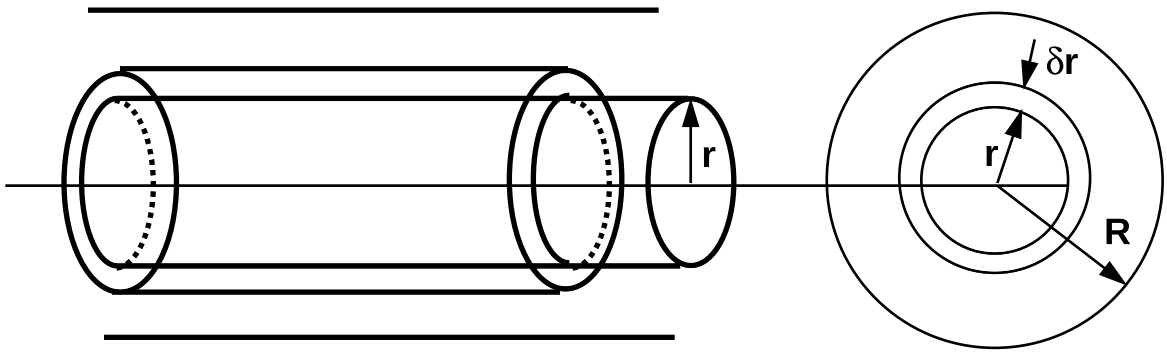
$$\tau = \mu \frac{du}{dr}$$
$$\frac{du}{dr} = -2 \times 16000r = -32000r$$
$$F = -0.00027 \times 32000 \times 0.0125 \times (2 \times \pi \times 0.0125)$$
$$= 8.48 \times 10^{-3}\, N$$

7.2

A liquid whose coefficient of viscosity is m flows below the critical velocity for laminar flow in a circular pipe of diameter d and with mean velocity u. Show that the pressure loss in a length of pipe is $32um/d^2$.
Oil of viscosity 0.05 kg/ms flows through a pipe of diameter 0.1m with a velocity of 0.6m/s. Calculate the loss of pressure in a length of 120m.
[11 520 N/m^2]

See the proof in the lecture notes for

Consider a cylinder of fluid, length L, radius r, flowing steadily in the centre of a pipe



The fluid is in equilibrium, shearing forces equal the pressure forces.

$$\tau\; 2\pi r\; L = \Delta p\; A = \Delta p \pi r^2$$

$$\tau = \frac{\Delta p}{L}\frac{r}{2}$$

Newtons law of viscosity $\tau = \mu \dfrac{du}{dy}$,

We are measuring from the pipe centre, so $\tau = -\mu \dfrac{du}{dr}$

Giving:

$$\frac{\Delta p}{L}\frac{r}{2} = -\mu \frac{du}{dr}$$

$$\frac{du}{dr} = -\frac{\Delta p}{L}\frac{r}{2\mu}$$

In an integral form this gives an expression for velocity,

$$u = -\frac{\Delta p}{L}\frac{1}{2\mu}\int r\, dr$$

The value of velocity at a point distance r from the centre

$$u_r = -\frac{\Delta p}{L}\frac{r^2}{4\mu} + C$$

At $r = 0$, (the centre of the pipe), $u = u_{max}$, at $r = R$ (the pipe wall) $u = 0$;

$$C = \frac{\Delta p}{L}\frac{R^2}{4\mu}$$

At a point r from the pipe centre when the flow is laminar:

$$u_r = \frac{\Delta p}{L}\frac{1}{4\mu}\left(R^2 - r^2\right)$$

The flow in an annulus of thickness δr

$$\delta Q = u_r A_{annulus}$$

$$A_{annulus} = \pi(r+\delta r)^2 - \pi r^2 \approx 2\pi r \delta r$$

$$\delta Q = \frac{\Delta p}{L}\frac{1}{4\mu}\left(R^2 - r^2\right)2\pi r \delta r$$

$$Q = \frac{\Delta p}{L}\frac{\pi}{2\mu}\int_0^R \left(R^2 r - r^3\right)\, dr$$

$$= \frac{\Delta p}{L}\frac{\pi R^4}{8\mu} = \frac{\Delta p\, \pi d^4}{L 128\mu}$$

So the discharge can be written

$$Q = \frac{\Delta p}{L}\frac{\pi d^4}{128\mu}$$

To get pressure loss in terms of the velocity of the flow, use the mean velocity:

$$u = Q / A$$

$$u = \frac{\Delta p\, d^2}{32\mu L}$$

$$\Delta p = \frac{32\mu L u}{d^2}$$

$$\Delta p = \frac{32\mu u}{d^2} \quad \text{per unit length}$$

b) From the question *μ= 0.05 kg/ms* *d = 0.1m*

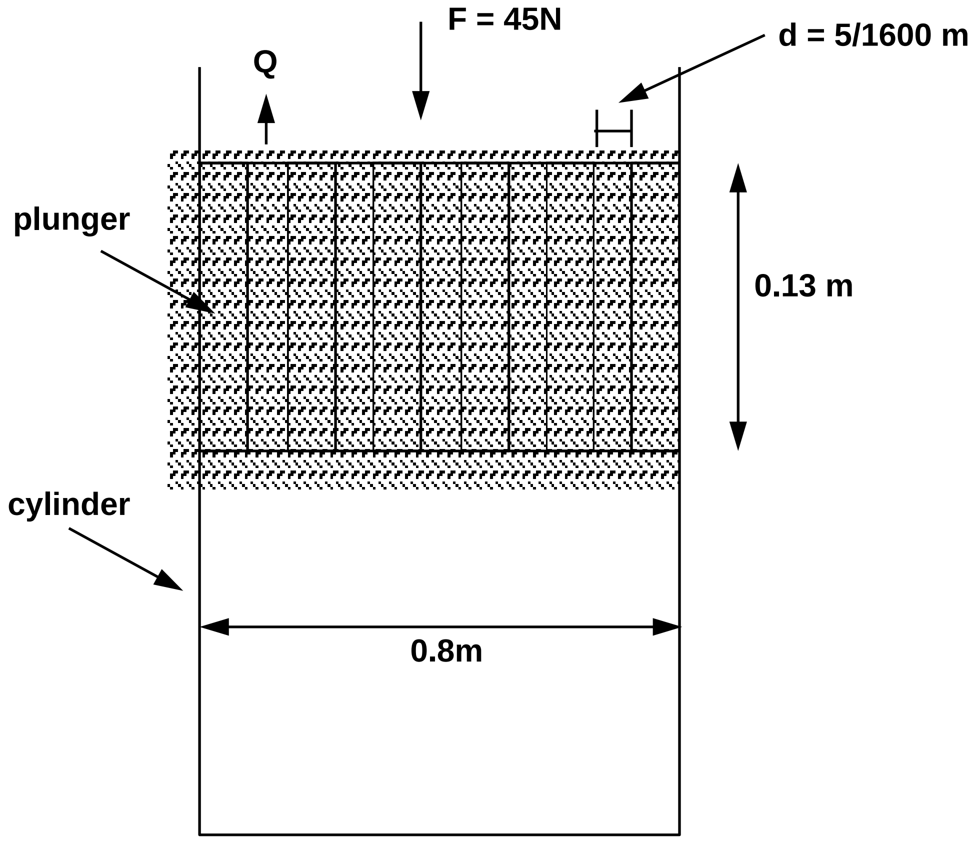
u = 0.6 m/s *L = 120.0m*

$$\Delta p = \frac{32 \times 0.05 \times 120 \times 0.6}{0.1^2} = 11520\, N / m^2$$

7.3

A plunger of 0.08m diameter and length 0.13m has four small holes of diameter 5/1600 m drilled through in the direction of its length. The plunger is a close fit inside a cylinder, containing oil, such that no oil is assumed to pass between the plunger and the cylinder. If the plunger is subjected to a vertical downward force of 45N (including its own weight) and it is assumed that the upward flow through the four small holes is laminar, determine the speed of the fall of the plunger. The coefficient of velocity of the oil is 0.2 kg/ms.

[0.00064 m/s]



Flow through each tube given by Hagen-Poiseuille equation

$$Q = \frac{\Delta p}{L}\frac{\pi d^4}{128\mu}$$

There are 4 of these so total flow is

$$Q = 4\frac{\Delta p}{L}\frac{\pi d^4}{128\mu} = \Delta p\frac{4\pi(5/1600)^4}{0.13\times128\times0.2} = \Delta p 3.601\times10^{-10}$$

$$Force = pressure \times area$$

$$F = 45 = \Delta p\left(\pi\left(\frac{0.08}{2}\right)^2 - 4\pi\left(\frac{5/1600}{2}\right)^2\right)$$

$$\Delta p = 9007.206\, N/m^2$$

$$Q = 3.24\times10^{-6}\, m^3/s$$

Flow up through piston = flow displaced by moving piston

$Q = Av_{piston}$

$3.24\times10^{-6} = \pi\times0.04^2\times v_{piston}$

$v_{piston} = 0.00064$ m/s

7.4
A vertical cylinder of 0.075 metres diameter is mounted concentrically in a drum of 0.076metres internal diameter. Oil fills the space between them to a depth of 0.2m. The rotque required to rotate the cylinder in the drum is 4Nm when the speed of rotation is 7.5 revs/sec. Assuming that the end effects are negligible, calculate the coefficient of viscosity of the oil.
[0.638 kg/ms]

From the question $r_1 = 0.076/2$ $r_2 = 0.075/2$ Torque $= 4Nm,\ L = 0.2m$

The velocity of the edge of the cylinder is:

$$u_{cyl} = 7.5 \times 2\pi r = 7.5\times2\times\pi\times0.0375 = 1.767\ m/s$$

$$u_{drum} = 0.0$$

Torque needed to rotate cylinder

$$T = \tau \times surface\ area$$
$$4 = \tau\left(2\pi r_2 \times L\right)$$
$$\tau = 2263.54\ N/m^2$$

Distance between cylinder and drum $= r_1 - r_2 = 0.038 - 0.0375 = 0.005m$

Using Newtons law of viscosity:

$$\tau = \mu\frac{du}{dr}$$
$$\frac{du}{dr} = \frac{1.767 - 0}{0.0005}$$
$$\tau = 2263.5 = \mu 3534$$
$$\mu = 0.64\ \ kg/ms \qquad (Ns/m^2)$$

Dimensional analysis

8.1
A stationary sphere in water moving at a velocity of 1.6m/s experiences a drag of 4N. Another sphere of twice the diameter is placed in a wind tunnel. Find the velocity of the air and the drag which will give dynamically similar conditions. The ratio of kinematic viscosities of air and water is 13, and the density of air 1.28 kg/m^3.
[10.4m/s 0.865N]

Draw up the table of values you have for each variable:

variable	water	air
u	1.6m/s	u_{air}
Drag	4N	D_{air}
ν	ν	13ν
ρ	1000 kg/m^3	1.28 kg/m^3
d	d	2d

Kinematic viscosity is dynamic viscosity over density = $\nu = \mu/\rho$.

The Reynolds number = $\mathrm{Re} = \dfrac{\rho u d}{\mu} = \dfrac{ud}{\nu}$

Choose the three recurring (governing) variables; u, d, ρ.

From Buckinghams π theorem we have m-n = 5 - 3 = 2 non-dimensional groups.

$$\phi(u, d, \rho, D, \nu) = 0$$
$$\phi(\pi_1, \pi_2) = 0$$
$$\pi_1 = u^{a_1} d^{b_1} \rho^{c_1} D$$
$$\pi_2 = u^{a_2} d^{b_2} \rho^{c_2} \nu$$

As each π group is dimensionless then considering the dimensions, for the first group, π_1:
(note D is a force with dimensions MLT^{-2})

$$M^0 L^0 T^0 = \left(LT^{-1}\right)^{a_1} (L)^{b_1} \left(ML^{-3}\right)^{c_1} MLT^{-2}$$

M] $0 = c_1 + 1$

$c_1 = -1$

L] $0 = a_1 + b_1 - 3c_1 + 1$

$-4 = a_1 + b_1$

T] $0 = -a_1 - 2$

$a_1 = -2$

$b_1 = -2$

$$\pi_1 = u^{-2} d^{-2} \rho^{-1} D$$
$$= \frac{D}{\rho u^2 d^2}$$

And the second group π_2 :

$$M^0 L^0 T^0 = \left(LT^{-1}\right)^{a_2} (L)^{b_2} \left(ML^{-3}\right)^{c_2} L^2 T^{-1}$$

M] $0 = c_2$

L] $0 = a_2 + b_2 - 3c_2 + 2$

$-2 = a_2 + b_2$

T] $0 = -a_2 - 1$

$a_2 = -1$

$b_2 = -1$

$$\pi_2 = u^{-1} d^{-1} \rho^0 \nu$$
$$= \frac{\nu}{ud}$$

So the physical situation is described by this function of nondimensional numbers,

$$\phi(\pi_1, \pi_2) = \phi\left(\frac{D}{\rho u^2 d^2}, \frac{\nu}{ud}\right) = 0$$

For dynamic similarity these non-dimensional numbers are the same for the both the sphere in water and in the wind tunnel i.e.

$$\pi_{1_{air}} = \pi_{1_{water}}$$
$$\pi_{2_{air}} = \pi_{2_{water}}$$

For π_1

$$\left(\frac{D}{\rho u^2 d^2}\right)_{air} = \left(\frac{D}{\rho u^2 d^2}\right)_{water}$$
$$\frac{D_{air}}{1.28 \times 10.4^2 \times (2d)^2} = \frac{4}{1000 \times 1.6^2 \times d^2}$$
$$D_{air} = 0.865\,N$$

For π_2

$$\left(\frac{\nu}{ud}\right)_{air} = \left(\frac{\nu}{ud}\right)_{water}$$
$$\frac{13\nu}{u_{air} \times 2d} = \frac{\nu}{1.6 \times d}$$
$$u_{air} = 10.4\,m/s$$

8.2

Explain briefly the use of the Reynolds number in the interpretation of tests on the flow of liquid in pipes. Water flows through a 2cm diameter pipe at 1.6m/s. Calculate the Reynolds number and find also the velocity required to give the same Reynolds number when the pipe is transporting air. Obtain the ratio of pressure drops in the same length of pipe for both cases. For the water the kinematic viscosity was 1.31×10^{-6} m^2/s and the density was 1000 kg/m^3. For air those quantities were 15.1×10^{-6} m^2/s and 1.19kg/m^3.

[24427, 18.4m/s, 0.157]

Draw up the table of values you have for each variable:

variable	water	air
u	1.6m/s	u_{air}
p	p_{water}	p_{air}
ρ	1000 kg/m^3	1.19kg/m^3
ν	1.31×10^{-6}m^2/s	15.1×10^{-6}m^2/s
ρ	1000 kg/m^3	1.28 kg/m^3
d	0.02m	0.02m

Kinematic viscosity is dynamic viscosity over density = $\nu = \mu/\rho$.

The Reynolds number = $\mathrm{Re} = \dfrac{\rho u d}{\mu} = \dfrac{ud}{\nu}$

Reynolds number when carrying water:

$$\mathrm{Re}_{water} = \frac{ud}{\nu} = \frac{1.6\times 0.02}{1.31\times 10^{-6}} = 24427$$

To calculate Re_{air} we know,

$$\begin{aligned}\mathrm{Re}_{water} &= \mathrm{Re}_{air}\\ 24427 &= \frac{u_{air}\,0.02}{15\times 10^{-6}}\\ u_{air} &= 18.44 m/s\end{aligned}$$

To obtain the ratio of pressure drops we must obtain an expression for the pressure drop in terms of governing variables.

Choose the three recurring (governing) variables; u, d, ρ.

From Buckinghams π theorem we have m-n = 5 - 3 = 2 non-dimensional groups.

$$\begin{aligned}\phi(u, d, \rho, \nu, p) &= 0\\ \phi(\pi_1, \pi_2) &= 0\\ \pi_1 &= u^{a_1} d^{b_1} \rho^{c_1} \nu\\ \pi_2 &= u^{a_2} d^{b_2} \rho^{c_2} p\end{aligned}$$

As each π group is dimensionless then considering the dimensions, for the first group, π_1:

$$M^0L^0T^0 = \left(LT^{-1}\right)^{a_1}\left(L\right)^{b_1}\left(ML^{-3}\right)^{c_1} L^2T^{-1}$$

M] $0 = c_1$

L] $0 = a_1 + b_1 - 3c_1 + 2$

$-2 = a_1 + b_1$

T] $0 = -a_1 - 1$

$a_1 = -1$

$b_1 = -1$

$$\pi_1 = u^{-1}d^{-1}\rho^0\nu$$
$$= \frac{\nu}{ud}$$

And the second group π_2 :

(note p is a pressure (force/area) with dimensions $ML^{-1}T^{-2}$)

$$M^0L^0T^0 = \left(LT^{-1}\right)^{a_1}\left(L\right)^{b_1}\left(ML^{-3}\right)^{c_1} MT^{-2}L^{-1}$$

M] $0 = c_2 + 1$

$c_2 = -1$

L] $0 = a_2 + b_2 - 3c_2 - 1$

$-2 = a_2 + b_2$

T] $0 = -a_2 - 2$

$a_2 = -2$

$b_2 = 0$

$$\pi_2 = u^{-2}\rho^{-1}p$$
$$= \frac{p}{\rho u^2}$$

So the physical situation is described by this function of nondimensional numbers,

$$\phi(\pi_1, \pi_2) = \phi\left(\frac{\nu}{ud}, \frac{p}{\rho u^2}\right) = 0$$

For dynamic similarity these non-dimensional numbers are the same for the both water and air in the pipe.

$$\pi_{1air} = \pi_{1water}$$
$$\pi_{2air} = \pi_{2water}$$

We are interested in the relationship involving the pressure i.e. π_2

$$\left(\frac{p}{\rho u^2}\right)_{air} = \left(\frac{p}{\rho u^2}\right)_{water}$$

$$\frac{p_{water}}{p_{air}} = \frac{\rho_{water} u_{water}^2}{\rho_{air} u_{air}^2}$$

$$= \frac{1000 \times 1.6^2}{1.19 \times 18.44^2} = \frac{1}{0.158} = 6.327$$

Show that Reynold number, ρud/μ, is non-dimensional. If the discharge Q through an orifice is a function of the diameter d, the pressure difference p, the density ρ, and the viscosity μ, show that $Q = Cp^{1/2}d^2/\rho^{1/2}$ where C is some function of the non-dimensional group $(d\rho^{1/2}d^{1/2}/\mu)$.
Draw up the table of values you have for each variable:

The dimensions of these following variables are

ρ	ML^{-3}
u	LT^{-1}
d	L
μ	$ML^{-1}T^{-1}$

$Re = ML^{-3}\ LT^{-1}L(ML^{-1}T^{-1})^{-1} = ML^{-3}\ LT^{-1}\ L\ M^{-1}LT = 1$

i.e. Re is dimensionless.

We are told from the question that there are 5 variables involved in the problem: d, p, ρ, μ and Q.

Choose the three recurring (governing) variables; Q, d, ρ.

From Buckinghams π theorem we have m-n = 5 - 3 = 2 non-dimensional groups.

$$\phi(Q, d, \rho, \mu, p) = 0$$

$$\phi(\pi_1, \pi_2) = 0$$

$$\pi_1 = Q^{a_1} d^{b_1} \rho^{c_1} \mu$$

$$\pi_2 = Q^{a_2} d^{b_2} \rho^{c_2} p$$

As each π group is dimensionless then considering the dimensions, for the first group, π_1:

$$M^0 L^0 T^0 = \left(L^3 T^{-1}\right)^{a_1} (L)^{b_1} \left(ML^{-3}\right)^{c_1} ML^{-1}T^{-1}$$

M] $0 = c_1 + 1$

$c_1 = -1$

L] $0 = 3a_1 + b_1 - 3c_1 - 1$

$-2 = 3a_1 + b_1$

T] $0 = -a_1 - 1$

$a_1 = -1$

$b_1 = 1$

$$\pi_1 = Q^{-1} d^1 \rho^{-1} \mu$$
$$= \frac{d\mu}{\rho Q}$$

And the second group π_2 :

(note p is a pressure (force/area) with dimensions $ML^{-1}T^{-2}$)

$$M^0 L^0 T^0 = \left(L^3 T^{-1}\right)^{a_1} (L)^{b_1} \left(ML^{-3}\right)^{c_1} MT^{-2} L^{-1}$$

M] $0 = c_2 + 1$

$c_2 = -1$

L] $0 = 3a_2 + b_2 - 3c_2 - 1$

$-2 = 3a_2 + b_2$

T] $0 = -a_2 - 2$

$a_2 = -2$

$b_2 = 4$

$$\pi_2 = Q^{-2} d^4 \rho^{-1} p$$
$$= \frac{d^4 p}{\rho Q^2}$$

So the physical situation is described by this function of non-dimensional numbers,

$$\phi(\pi_1, \pi_2) = \phi\left(\frac{d\mu}{Q\rho}, \frac{d^4 p}{\rho Q^2}\right) = 0$$

or

$$\frac{d\mu}{Q\rho} = \phi_1\left(\frac{d^4 p}{\rho Q^2}\right)$$

The question wants us to show : $Q = f\left(\frac{d\rho^{1/2} p^{1/2}}{\mu}\right)\left(\frac{d^2 p^{1/2}}{\rho}\right)$

Take the reciprocal of square root of π_2: $\frac{1}{\sqrt{\pi_2}} = \frac{\rho^{1/2} Q}{d^2 p^{1/2}} = \pi_{2a}$,

Convert π_1 by multiplying by this number

$$\pi_{1a} = \pi_1 \pi_{2a} = \frac{d\mu}{Q\rho} \frac{\rho^{1/2} Q}{d^2 p^{1/2}} = \frac{\mu}{d\rho^{1/2} p^{1/2}}$$

then we can say

$$\phi\left(1/\pi_{1a},\pi_{2a}\right)=\phi\left(\frac{p^{1/2}\rho^{1/2}d}{\mu},\frac{d^2p^{1/2}}{\rho^{1/2}}\right)=0$$

or

$$Q=\phi\left(\frac{p^{1/2}\rho^{1/2}d}{\mu}\right)\frac{d^2p^{1/2}}{\rho^{1/2}}$$

8.4

A cylinder 0.16m in diameter is to be mounted in a stream of water in order to estimate the force on a tall chimney of 1m diameter which is subject to wind of 33m/s. Calculate (A) the speed of the stream necessary to give dynamic similarity between the model and chimney, (b) the ratio of forces.

Chimney: ρ = 1.12kg/m^3 $\mu = 16\times10^{-6}$ kg/ms

Model: ρ = 1000kg/m^3 $\mu = 8\times10^{-4}$ kg/ms

[11.55m/s, 0.057]

Draw up the table of values you have for each variable:

variable	water	air
u	u_{water}	33m/s
F	F_{water}	F_{air}
ρ	1000 kg/m^3	1.12kg/m^3
μ	8×10^{-4} kg/ms	16×10^{-6}kg/ms
d	0.16m	1m

Kinematic viscosity is dynamic viscosity over density = $\nu = \mu/\rho$.

The Reynolds number = $\mathrm{Re}=\frac{\rho ud}{\mu}=\frac{ud}{\nu}$

For dynamic similarity:

$$\mathrm{Re}_{water}=\mathrm{Re}_{air}$$
$$\frac{1000u_{water}0.16}{8\times10^{-4}}=\frac{1.12\times33\times1}{16\times10^{-6}}$$
$$u_{water}=11.55m/s$$

To obtain the ratio of forces we must obtain an expression for the force in terms of governing variables.

Choose the three recurring (governing) variables; u, d, ρ, F, μ.

From Buckinghams π theorem we have m-n = 5 - 3 = 2 non-dimensional groups.

$$\phi\left(u,d,\rho,\mu,F\right)=0$$
$$\phi\left(\pi_1,\pi_2\right)=0$$
$$\pi_1=u^{a_1}d^{b_1}\rho^{c_1}\mu$$
$$\pi_2=u^{a_2}d^{b_2}\rho^{c_2}F$$

As each π group is dimensionless then considering the dimensions, for the first group, π_1:

$$M^0 L^0 T^0 = \left(LT^{-1}\right)^{a_1} (L)^{b_1} \left(ML^{-3}\right)^{c_1} ML^{-1}T^{-1}$$

M] $0 = c_1 + 1$

$c_1 = -1$

L] $0 = a_1 + b_1 - 3c_1 - 1$

$-2 = a_1 + b_1$

T] $0 = -a_1 - 1$

$a_1 = -1$

$b_1 = -1$

$$\pi_1 = u^{-1} d^{-1} \rho^{-1} \mu$$
$$= \frac{\mu}{\rho u d}$$

i.e. the (inverse of) Reynolds number

And the second group π_2 :

$$M^0 L^0 T^0 = \left(LT^{-1}\right)^{a_2} (L)^{b_2} \left(ML^{-3}\right)^{c_2} ML^{-1}T^{-2}$$

M] $0 = c_2 + 1$

$c_2 = -1$

L] $0 = a_2 + b_2 - 3c_2 - 1$

$-3 = a_2 + b_2$

T] $0 = -a_2 - 2$

$a_2 = -2$

$b_2 = -1$

$$\pi_2 = u^{-2} d^{-1} \rho^{-1} F$$
$$= \frac{F}{u^2 d \rho}$$

So the physical situation is described by this function of nondimensional numbers,

$$\phi(\pi_1, \pi_2) = \phi\left(\frac{\mu}{\rho u d}, \frac{F}{\rho d u^2}\right) = 0$$

For <u>dynamic similarity</u> these non-dimensional numbers are the same for the both water and air in the pipe.

$$\pi_{1_{air}} = \pi_{1_{water}}$$
$$\pi_{2_{air}} = \pi_{2_{water}}$$

To find the ratio of forces for the different fluids use π_2

$$\pi_{2\,air} = \pi_{2\,water}$$

$$\left(\frac{F}{\rho u^2 d}\right)_{air} = \left(\frac{F}{\rho u^2 d}\right)_{water}$$

$$\left(\frac{F}{\rho u^2 d}\right)_{air} = \left(\frac{F}{\rho u^2 d}\right)_{water}$$

$$\frac{F_{air}}{F_{water}} = \frac{1.12 \times 33^2 \times 1}{1000 \times 11.55^2 \times 0.16} = 0.057$$

8.5

If the resistance to motion, R, of a sphere through a fluid is a function of the density ρ and viscosity μ of the fluid, and the radius r and velocity u of the sphere, show that R is given by

$$R = \frac{\mu^2}{\rho} f\left(\frac{\rho u r}{\mu}\right)$$

Hence show that if at very low velocities the resistance R is proportional to the velocity u, then R = kμru where k is a dimensionless constant.
A fine granular material of specific gravity 2.5 is in uniform suspension in still water of depth 3.3m. Regarding the particles as spheres of diameter 0.002cm find how long it will take for the water to clear. Take k=6π and μ=0.0013 kg/ms.
[218mins 39.3sec]

Choose the three recurring (governing) variables; u, r, ρ, R, μ.

From Buckinghams π theorem we have m-n = 5 - 3 = 2 non-dimensional groups.

$$\phi(u, r, \rho, \mu, R) = 0$$

$$\phi(\pi_1, \pi_2) = 0$$

$$\pi_1 = u^{a_1} r^{b_1} \rho^{c_1} \mu$$

$$\pi_2 = u^{a_2} r^{b_2} \rho^{c_2} R$$

As each π group is dimensionless then considering the dimensions, for the first group, π_1:

$$M^0 L^0 T^0 = \left(LT^{-1}\right)^{a_1} (L)^{b_1} \left(ML^{-3}\right)^{c_1} ML^{-1}T^{-1}$$

M] $0 = c_1 + 1$

$c_1 = -1$

L] $0 = a_1 + b_1 - 3c_1 - 1$

$-2 = a_1 + b_1$

T] $0 = -a_1 - 1$

$a_1 = -1$

$b_1 = -1$

$$\pi_1 = u^{-1} r^{-1} \rho^{-1} \mu$$

$$= \frac{\mu}{\rho u r}$$

i.e. the (inverse of) Reynolds number

And the second group π_2 :

$$M^0 L^0 T^0 = \left(LT^{-1}\right)^{a_2} (L)^{b_2} \left(ML^{-3}\right)^{c_2} ML^{-1}T^{-2}$$

M] $0 = c_2 + 1$

$c_2 = -1$

L] $0 = a_2 + b_2 - 3c_2 - 1$

$-3 = a_2 + b_2$

T] $0 = -a_2 - 2$

$a_2 = -2$

$b_2 = -1$

$$\pi_2 = u^{-2} r^{-1} \rho^{-1} R = \frac{R}{u^2 r \rho}$$

So the physical situation is described by this function of nondimensional numbers,

$$\phi(\pi_1, \pi_2) = \phi\left(\frac{\mu}{\rho u r}, \frac{R}{\rho r u^2}\right) = 0$$

or

$$\frac{R}{\rho r u^2} = \phi_1\left(\frac{\mu}{\rho u r}\right)$$

he question asks us to show $R = \frac{\mu^2}{\rho} f\left(\frac{\rho u r}{\mu}\right)$ or $\frac{R\rho}{\mu^2} = f\left(\frac{\rho u r}{\mu}\right)$

Multiply the LHS by the square of the RHS: (i.e. $\pi_2 \times (1/\pi_1^2)$)

$$\frac{R}{\rho r u^2} \times \frac{\rho^2 u^2 r^2}{\mu^2} = \frac{R\rho}{\mu^2}$$

So

$$\frac{R\rho}{\mu^2} = f\left(\frac{\rho u r}{\mu}\right)$$

The question tells us that R is proportional to u so the function *f* must be a constant, *k*

$$\frac{R\rho}{\mu^2} = k \frac{\rho u r}{\mu}$$

$$R = \mu k r u$$

The water will clear when the particle moving from the water surface reaches the bottom.

At terminal velocity there is no acceleration - the force R = mg - upthrust.

From the question:

$\sigma = 2.5$ so $\rho = 2500 \text{kg/m}^3$ $\mu = 0.0013$ kg/ms $k = 6\pi$

r = 0.00001m depth = 3.3m

$$mg = \frac{4}{3}\pi 0.00001^3 \times 9.81 \times (2500 - 1000)$$
$$= 6.16 \times 10^{-11}$$

$$\mu kru = 0.0013 \times 6\pi \times 0.00001u = 6.16 \times 10^{-11}$$
$$u = 2.52 \times 10^{-4} m/s$$
$$t = \frac{3.3}{2.52 \times 10^{-4}} = 218 \min\ 39.3 \sec$$